Optimizing quantum resources for enhanced measurement precision in noisy environments

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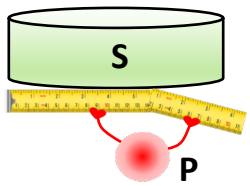
1. Abstract

Quantum resources like entanglement, coherence, and quantum correlations, are advanced techniques to enhance metrology precision. Here we introduce a variational approach to optimize the quantum resources and apply it to quantum metrology.

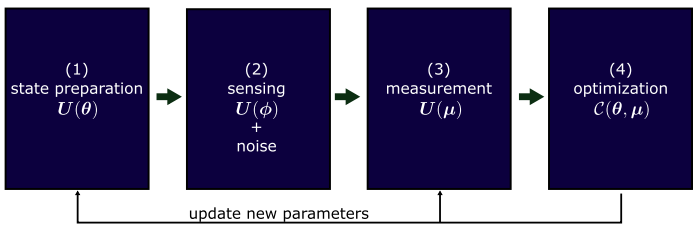
2. Quantum metrology



- System (s) + Probe (P)
- P imprints S information
- Readout P outcome
- Estimate information
- Challenge: how to improve the estimated precision?

3. Variational quantum metrology

Scheme



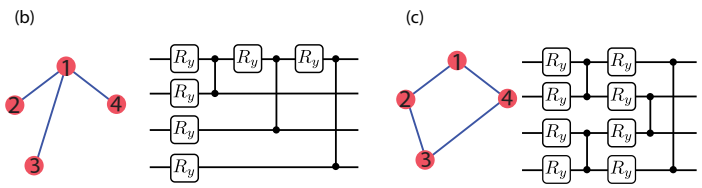
(1) prepare an ansatz for state preparation

(2) apply phase and noise

(3) apply measurement ansatz for POVM

(4) optimize the cost function and update

Graph-based Ansatz



Cost function

$$C(\boldsymbol{\theta},\boldsymbol{\mu}) = 1 - \frac{tr\left[\boldsymbol{Q}^{-1}\right]}{tr\left[\boldsymbol{F}^{-1}\right]}$$

4. Quantum estimation theory

Consider the interaction between a probe and external magnetic field

$$H(\boldsymbol{\phi}) = \phi_x J_x + \phi_y J_y + \phi_z J_z$$

Noise is given via Kraus operators

$$K_0(t) = \mathrm{diag}\left(1, \sqrt{1-q(t)}\right), K_1(t) = \mathrm{diag}\left(0, \sqrt{q(t)}\right),$$

$$q(t) = \begin{cases} 1-\exp(-\gamma t) & \text{Markovian,} \\ 1-\exp(-\frac{\gamma t^2}{2\tau_c}) & \text{non-Markovian.} \end{cases}$$

Quantum state

$$\rho(\boldsymbol{\phi},\gamma) = \sum_{k=0}^{1} K_k(t)\rho(\boldsymbol{\phi})K_k(t)^\dagger$$

5. Results

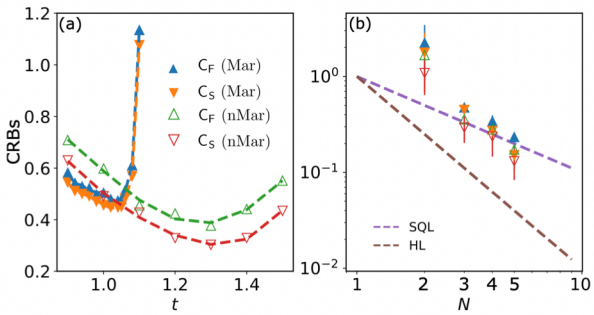


Fig. Plot of Cramer-Rao bound vs time and *N*

- the optimal classical and quantum bounds for Markovian and non-Markovian noises
- there exists an optimal sensing time for each case
- it can beat the standard quantum limit (SQL) for larger *N*

6. Conclusion

- We discussed the variational quantum resources for metrology.
- We examined the role of these resources for multiphase metrology under noise.

[1] Phu Trong Nguyen, Trung Kien Le, Hung Q Nguyen, Le Bin Ho, arXiv:2311.18225 (2023)

[2] Trung Kien Le, Hung Q Nguyen, Le Bin Ho, Scientific Reports 13, 17775 (2023).