Realizing Quantum Algorithms on Qubits Embedded into Trapped-Ion Qudits

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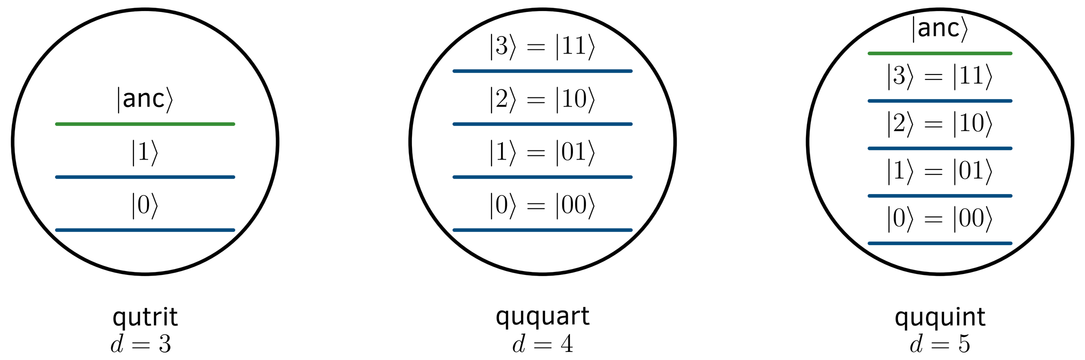
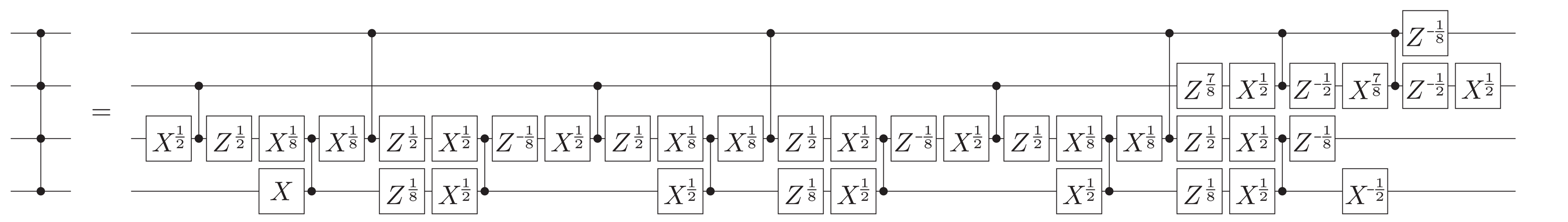
Qudits for quantum computing

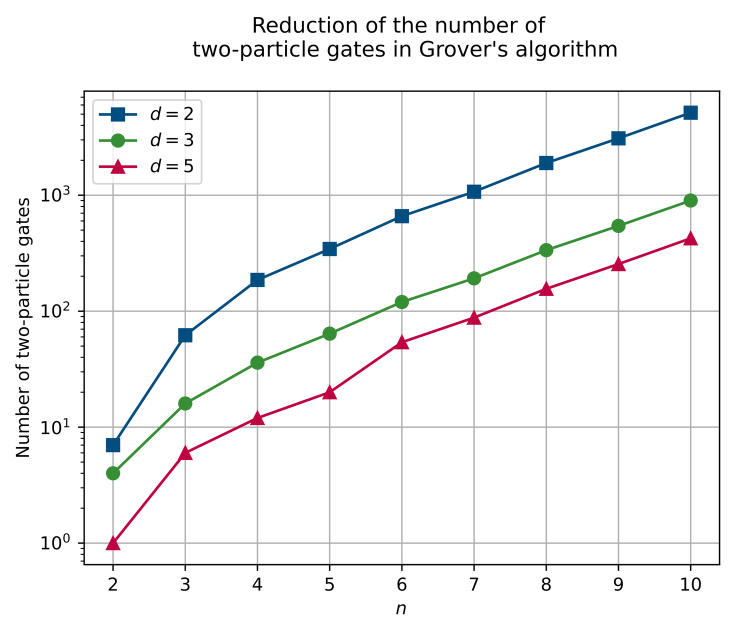
- Physical systems, used as qubits in quantum computing, have naturally more than 2 levels. Therefore, they can be considered as d-level quantum systems – qudits.



- Two prototypes of qudit-based processors on trapped-ions were presented last year [1,2].
- Like qubit-based ones, they operate with single- and two-particle gates, and the fidelity of the two-particle gates is lower.
- Substantial number of two-particle gates appears in multi-qubit gate decompositions.
- Best known 4-qubit C^3X gate decomposition [3] requires 14 two-qubit gates.



- Qudits allow reducing the number of two-particle gates in the realization of qubit circuits, if we suppose that:
  - Qutrit = qubit + ancillary level
  - Ququart = 2 qubits
  - Ququint = 2 qubits + ancillary level
  - Quhex = 2 qubits + 2 ancillary levels
  - Qusept = 2 qubits + 3 ancillary levels
  - Quoct = 3 qubits



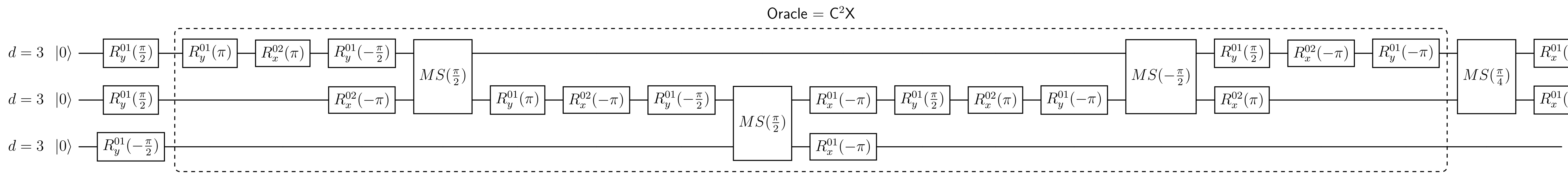
- In this work, we develop a set of methods for compiling quantum algorithms on qubits embedded in trapped-ion qudits with $d \in \{3, \ldots, 8\}$.

Basic gates for ion-based qudits

- Single-qudit gates:
$$R_\varphi^{\alpha,\beta}(\theta) = \exp(-\imath\theta\sigma_\varphi^{\alpha,\beta}/2)$$
where $\varphi \in \{x, y, z\}$, $\sigma_\varphi^{\alpha,\beta}$ – extended Pauli matrices, and α, β indicate transition in the qudit, i.e. $\sigma_x^{02} = |0\rangle\langle 2| + |2\rangle\langle 0|$.
- Two-qudit Mølmer–Sørensen gate:
$$\text{MS}(\chi) = \exp\left(-\imath[\sigma_x^{0,1} \otimes \sigma_x^{0,1}]\chi\right)$$
- Due to additional phases on higher levels, standard qubit decompositions cannot be used in a straightforward way.

Algorithms on qubits embedded into trapped-ion qutrits

- Single-qubit gates are implemented in the qubit subspace of a qutrit.
- Two-qubit gates are implemented as a sequence of MS(χ) and $R_\varphi^{\alpha,\beta}(\theta)$ gates $(\alpha, \beta \in \{0, 1\})$.
- The third level of a qutrit is used only for multi-qubit gate decompositions.
- N-qubit Toffoli gate $C^{N-1}X$ can be realized with $2N-3$ MS$(\pi/2)$ gates [4].



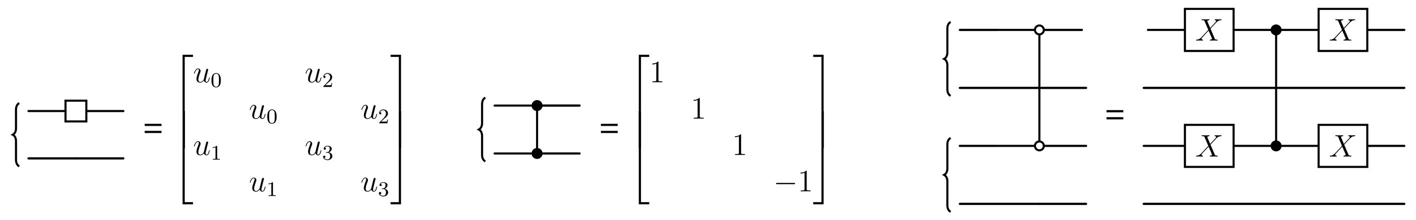
Grover's algorithm circuit for finding $\omega = 11$.

Circuits on qubits embedded into ququarts

- MS$(\chi) \to \exp\left(-\imath[\sigma_z^{0,1} \otimes \sigma_z^{0,1}]\chi\right) \xrightarrow{d=4}$

$$\begin{bmatrix} e^{-\imath\chi} &&&&&&&& \\ & e^{\imath\chi} &&&&&&& \\ && 1 &&&&&& \\ &&& 1 &&&&& \\ &&&& e^{\imath\chi} &&&& \\ &&&&& e^{-\imath\chi} &&& \\ &&&&&& 1 && \\ &&&&&&& \ddots & \\ &&&&&&&& 1 \end{bmatrix} \xrightarrow{\chi=\pi} \begin{bmatrix} -1 &&&&&&&& \\ & -1 &&&&&&& \\ && 1 &&&&&& \\ &&& 1 &&&&& \\ &&&& -1 &&&& \\ &&&&& -1 &&& \\ &&&&&& 1 && \\ &&&&&&& \ddots & \\ &&&&&&&& 1 \end{bmatrix}$$

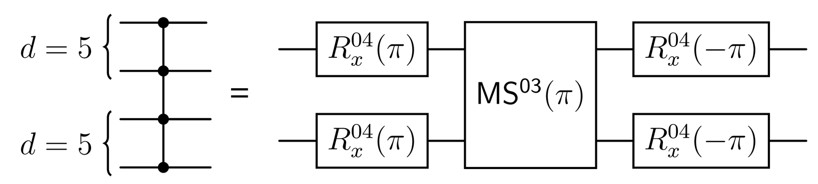
- Native MS(π) operation on ququarts corresponds to two-qubit gate between two qubits, located in different qudits:



- On the basis of MS(π) with $R_\varphi^{\alpha,\beta}(\theta)$ gates CZ gate between arbitrary pair of qubits in different ququarts can be obtained.
- CZ gate between two qubits in a singe ququart is implemented as a sequence of singe-qudit $R_\varphi^{\alpha,\beta}(\theta)$ gates.
- Knowing how to implement CZ gate between any pair of qubits in ququarts, standard qubit decomposition of multi-qubit gates can be used as a template.
- An ability to realize arbitrary single-qubit gates and two-qubit CZ gate in every qubits' pair provides the basis for the universal quantum computation.
- 4-qubit Toffoli gate can be implemented with 6 two-ququart MS(π) gates, if decomposition [3] is used as a template.

Circuits on qubits in high-dimensional qudits

- 5-level and 6-level qudits' spaces can be considered as a space of two qubits with 1 and 2 ancillary levels correspondingly.
- Single- and two-qubit gates in this case are implemented similarly to the ququart case.
- C^3Z gate can be realized via a single two-ququint MS$^{03}(\pi)$ gate.



- The highest $|4\rangle = |\text{anc}_0\rangle$ and $|5\rangle = |\text{anc}_1\rangle$ levels of a 6-level qudit can be used as ancillary states to provide ladder-like decomposition of a N-qubit gate with $N-3$ or $N-2$ two-qudit MS(π) gates.
- Two-quoct $(d = 8)$ gate MS(π) with single-qudit rotations realize 4-qubit C^3Z gate on two pairs of qubits located in different quocts.

Conclusion & Outlook

- We develop a set of methods for the realization of qubit quantum algorithms on ion-based qudits of various dimensions.
- The main feature of our methods is the use of two-qudit MS(χ) and single-qudit $R_\varphi^{\alpha,\beta}(\theta)$ gates in the multi-qubit gate decompositions, which are directly executed on the trapped-ion platform.

References

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