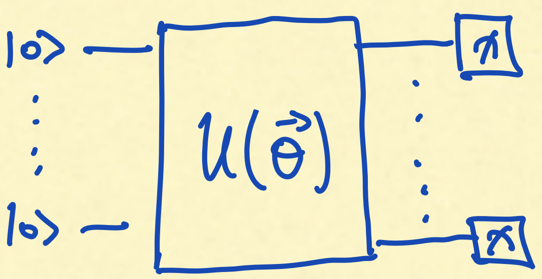
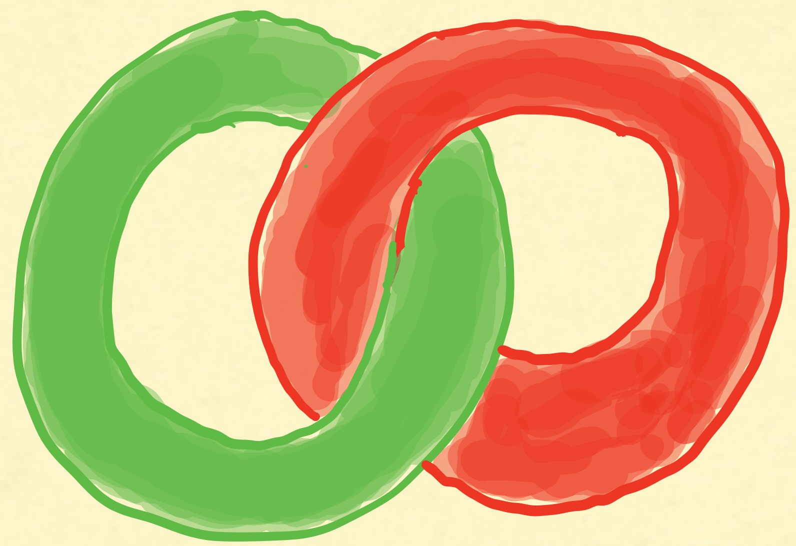
Circuit depth vs. energy for topological order in 2D

(joint w/ Isaac Kim)



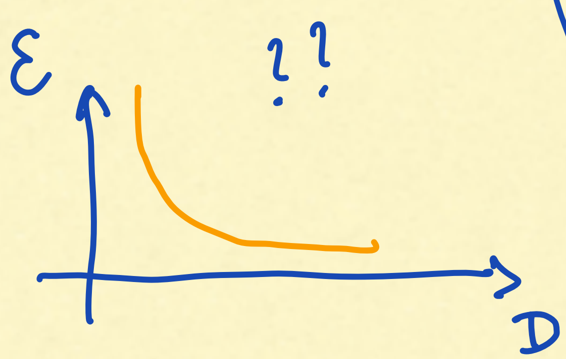
$$\min_{\vec\theta} \langle \psi(\vec\theta) | H | \psi(\vec\theta) \rangle$$

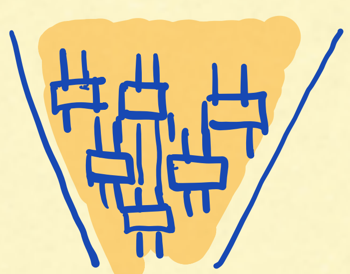
$$H_\Lambda = \sum_{X \subset \Lambda} h_X$$



$$\mathrm{Tr}\left[H_R \rho_\Lambda^\epsilon\right] \leq |R|\epsilon$$

$$\left\| \rho_\Lambda^\epsilon - \frac{\Pi_R^{TC} \rho_\Lambda^\epsilon \Pi_R^{TC}}{\mathrm{Tr}\left[\Pi_R^{TC} \rho_\Lambda^\epsilon \Pi_R^{TC}\right]} \right\|_1 \leq O(|R|\epsilon)$$



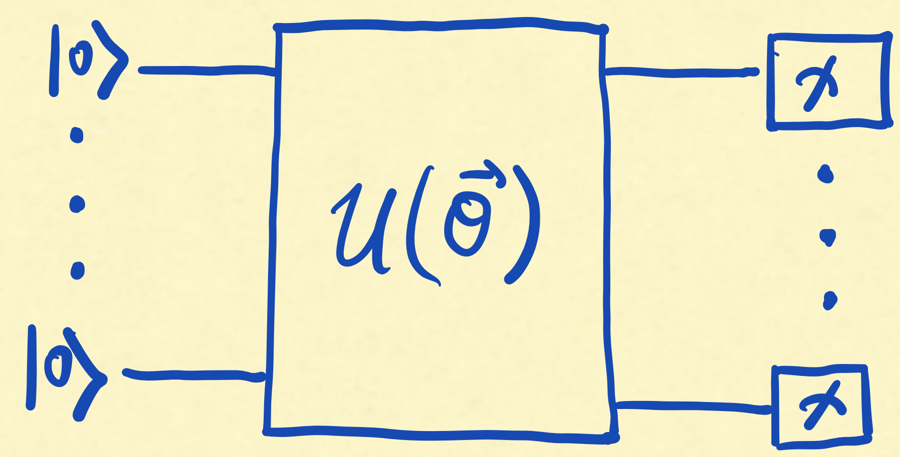


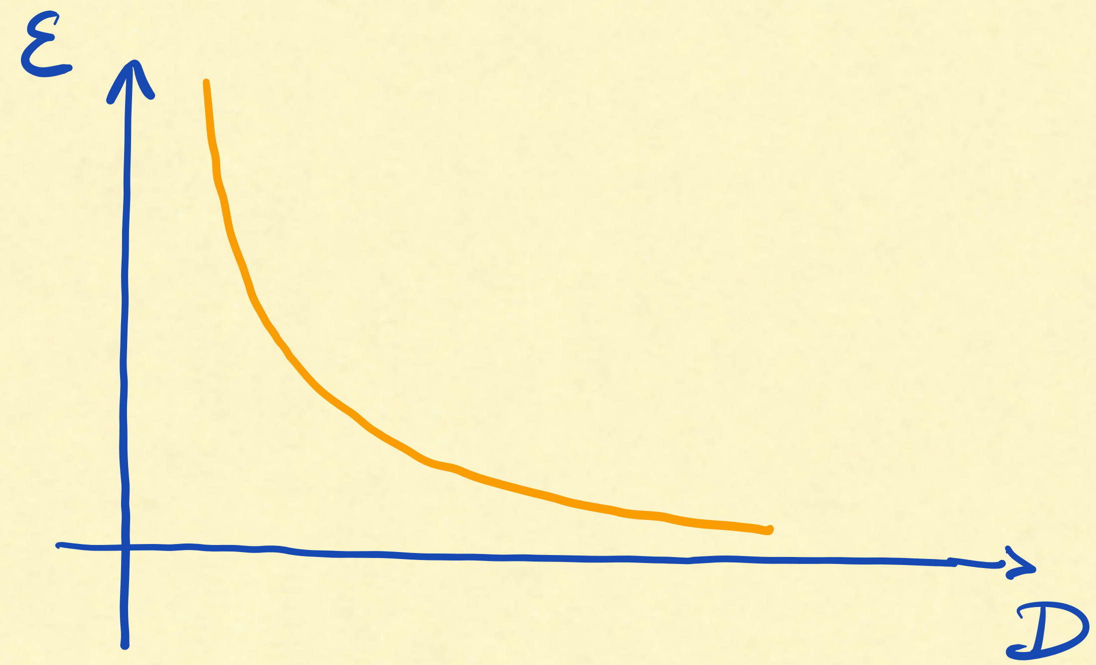
$$\langle \psi | P \otimes Q | \psi \rangle \neq \langle \psi | P | \psi \rangle \langle \psi | Q | \psi \rangle$$

ABSTRACT

- HARDNESS RESULT ON PREPARING LOW-ENERGY STATES OF TOPOLOGICALLY ORDERED MODELS IN 2D.
- MOTIVATED BY NUMERICAL WORK ON VARIATIONAL QUANTUM ALGORITHMS

→ In general, no exponential decay of energy density with circuit depth for VQE !





OUTLINE

- NISQ and the VQE
- Main result
- Proof sketch

LISTEN !

- Low energy states
- A witness for topological order
- Bounds on the witness

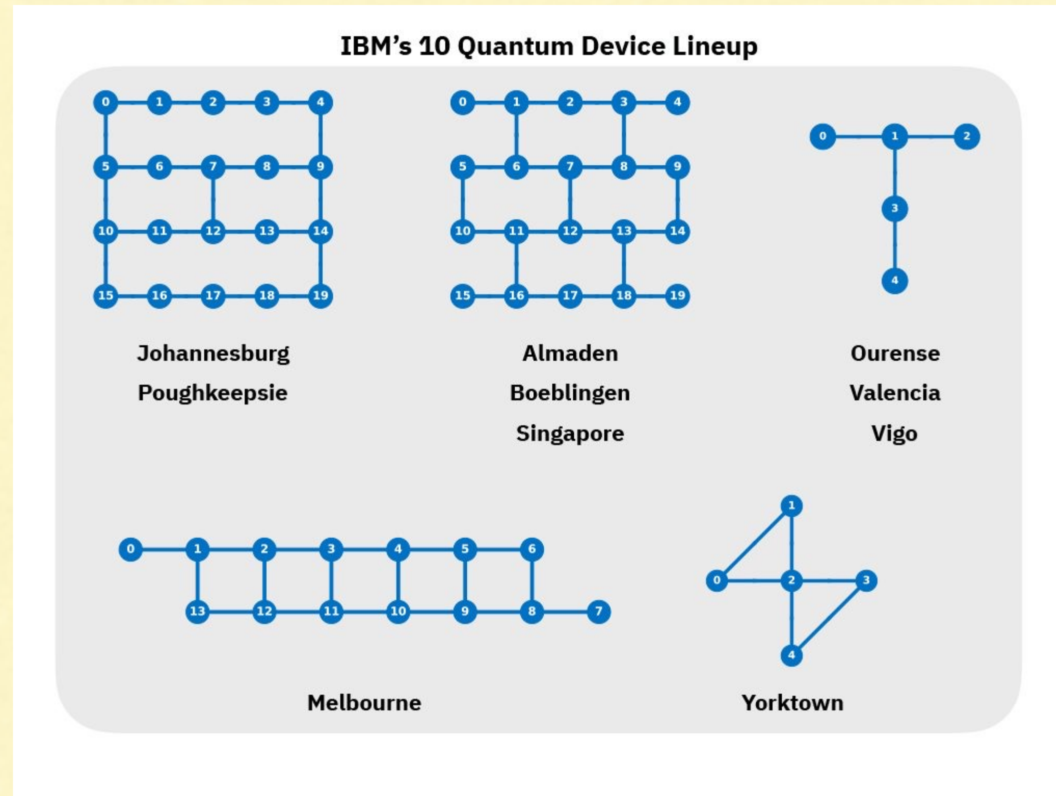
SLEEP !

- Discussion

WAKE UP !

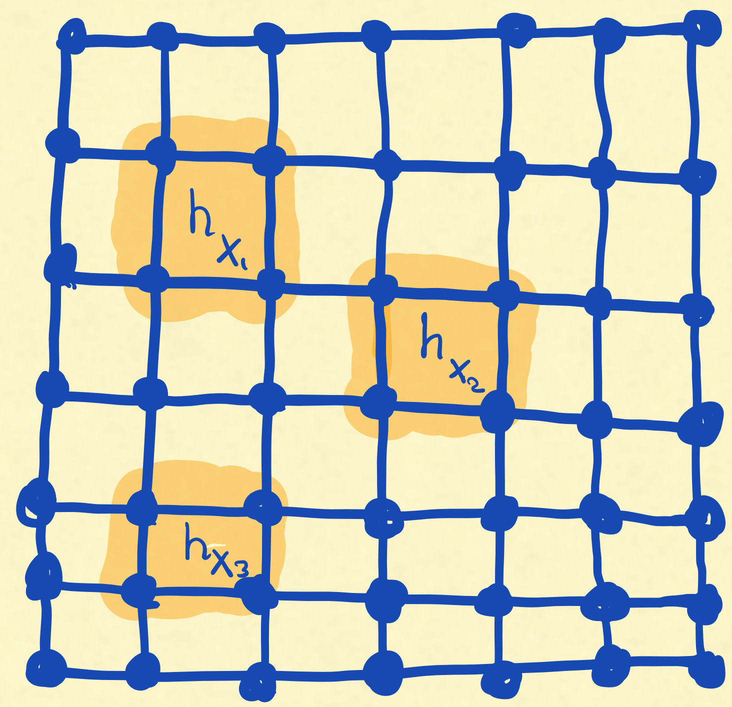
Estimating ground energies on a near-term device

- **Current devices are very limited:**
  - few qubits
  - very noisy
  - limited connectivity (2D nearest neighbor)
  - no adaptive circuits (yet!)



What can we do with them ???

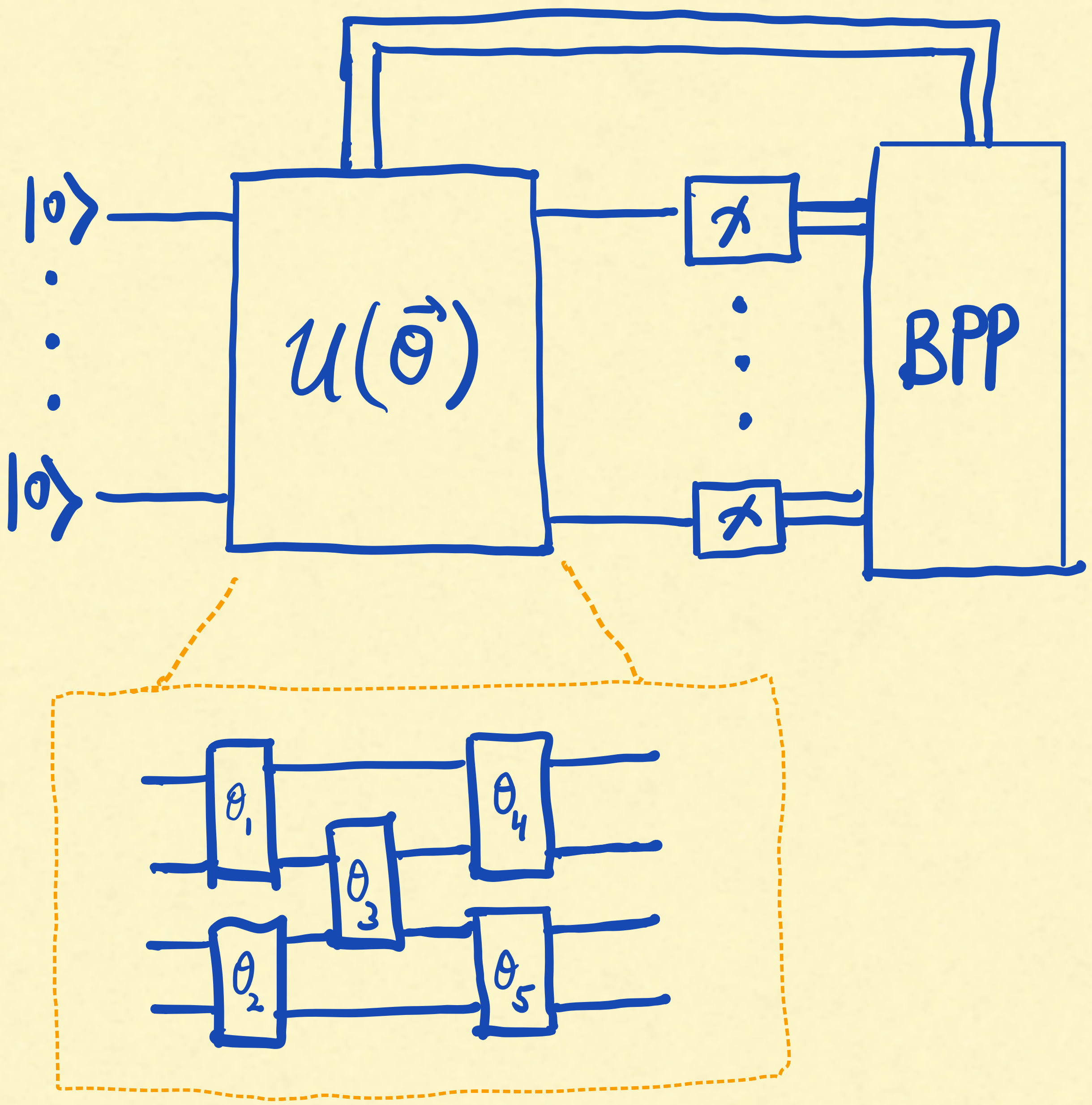
↳ Try to prepare groundstates of gapped lattice models!!



$$H_\Lambda = \sum_{X \subset \Lambda} h_X$$

Estimating ground energies on a near-term device

Popular paradigm: Variational quantum eigensolver (VQE)

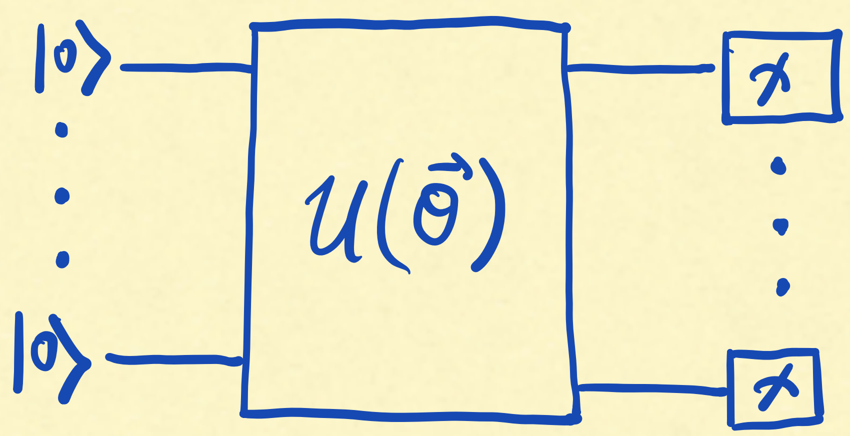


Algorithm: For $H = \sum_{e} \alpha_e P_e$

compute $\min_{\vec{\theta}} \langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle$

1) Initialize $|0\rangle^{\otimes N}$
2) Apply variational ansatz $U(\vec{\theta})$
3) Measure trial state $|\psi(\vec{\theta})\rangle$
4) Evaluate $\langle \psi(\vec{\theta}) | H | \psi(\vec{\theta}) \rangle$
5) Update $\vec{\theta}$
6) Repeat

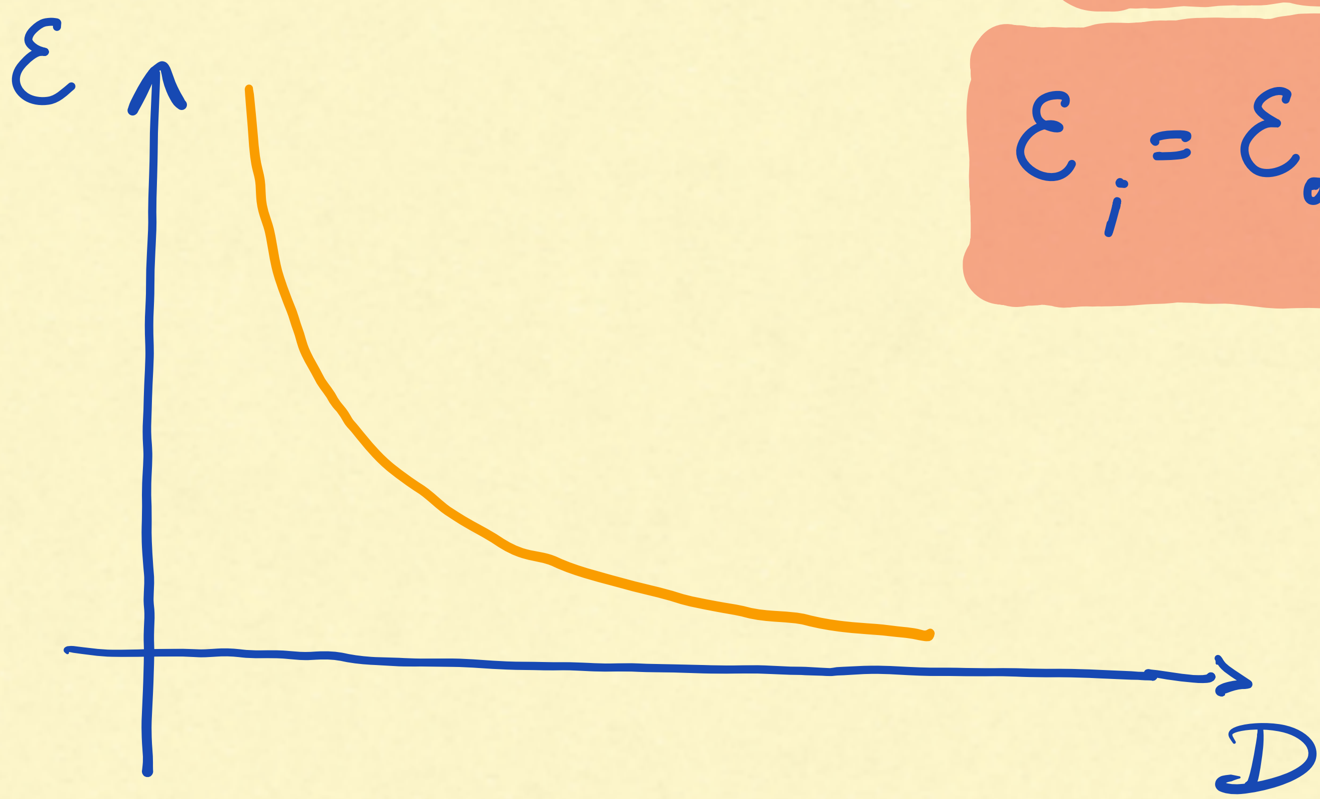
Estimating ground energies on a near-term device



to go below target energy density ε
how deep must our ansatz circuit be ?

polynomial decay vs. exponential decay

$\varepsilon_i = \varepsilon_0 + 1/\mathrm{poly}(D_i)$ vs. $\varepsilon_i = \varepsilon_0 + 1/\exp(D_i)$



Which one can we expect for interesting lattice models ?

Problem statement

Assume: • Topologically ordered, frustration-free 2D lattice Hamiltonian $H_\Lambda = \sum_{X \subset \Lambda} h_X$

• geometrically local gates in 2D (no adaptive gates!)

Task: Prepare state ρ_Λ^ε with global energy density at most ε, i.e.

$$\boxed{\mathrm{Tr}\left[H_\Lambda \rho_\Lambda^\varepsilon\right] \leq |\Lambda| \varepsilon}$$

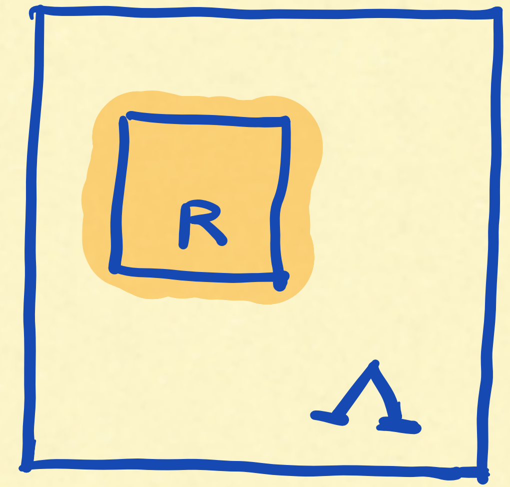
Main result

Worst case: circuit depth $D = \Omega\left(\mathrm{poly}(1/\varepsilon)\right)$

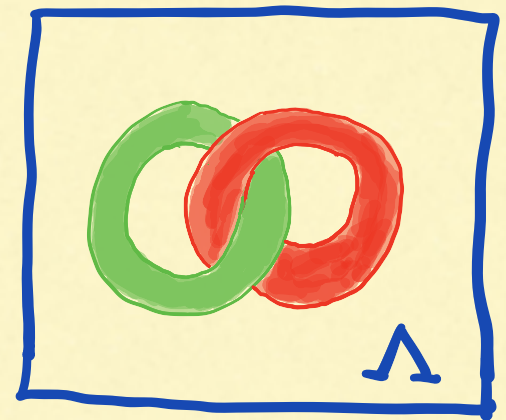
(for mixed ρ_Λ^ε, prepare purification $|\psi^\varepsilon\rangle_{\Lambda'}$ in depth D)

Proof sketch (high-level)

ρ_Λ^ε



1) Show existence of "good" patch R : ρ_R^ε close to ground space of H_R (H_Λ restricted to R)



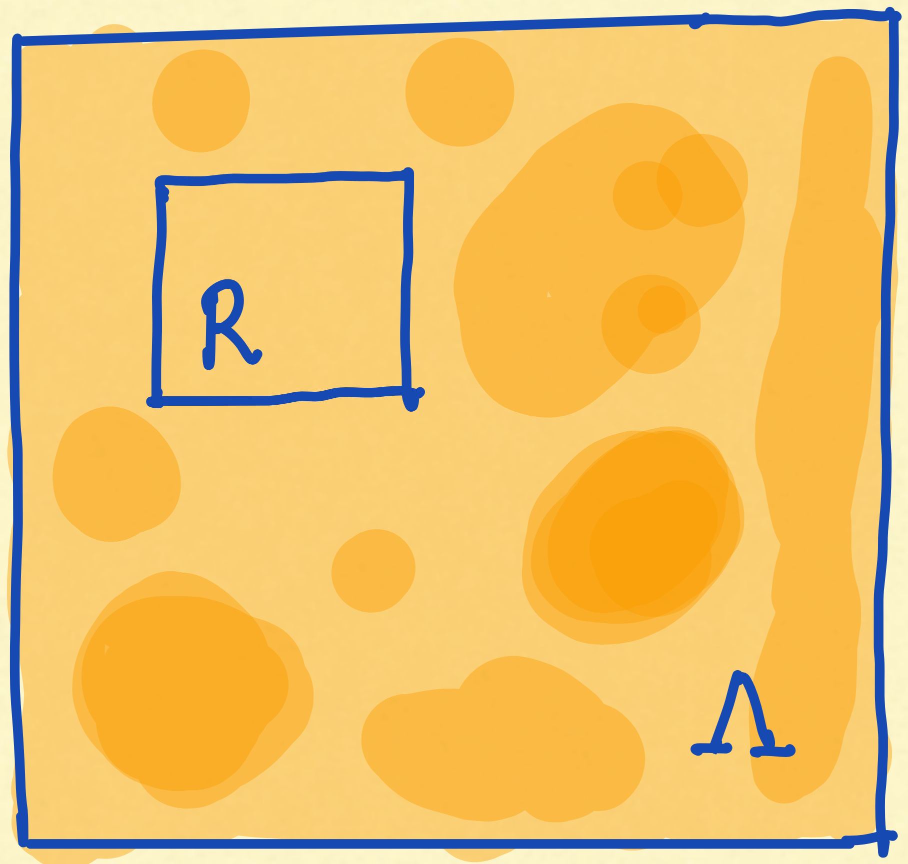
2) Generalize witness function $C(P,Q)_{|\psi\rangle}$ for topological order related to anyon braiding (see Haah (2013))

3) Put bounds on $C(P,Q)_{|\psi\rangle}$ on patch R for $|\psi^\varepsilon\rangle_\Lambda$.

$$\boxed{f(1/\varepsilon) \le C(P,Q)_{|\psi^\varepsilon\rangle} \le g(D)} \longrightarrow \boxed{D = \Omega\left(\sqrt{1/\varepsilon}\right)}$$

Main result!!

Low-energy states have ε-good subsystems



We show: There always exists subsystem with high overlap with ground state.

ε-good subsystem R:

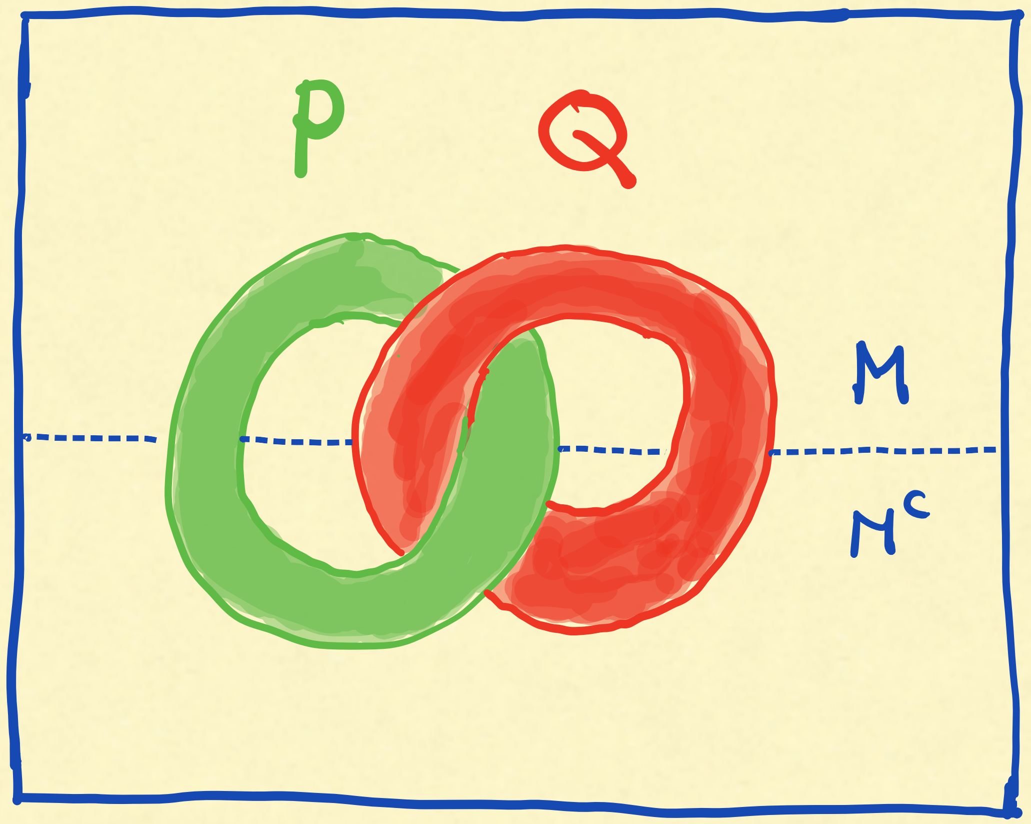
$$\boxed{\mathrm{Tr}\left[H_R \rho_R^{\varepsilon}\right] \leq |R|\,\varepsilon}$$

Lemma: For ε-good subsystem R:

$$\boxed{\left\| \rho_\Lambda^{\varepsilon} - \Pi_R^{0} \rho_\Lambda^{\varepsilon} \Pi_R^{0} \right\|_1 \leq \sqrt{|R|\,\varepsilon}}$$

A witness for topological order

Twist product



$$P \infty Q := \sum_{j,k} P^j_M Q^k_M \otimes Q^k_{M^c} P^j_{M^c}$$

w/ $P = \sum_j P^j_M \otimes P^j_{M^c}$

$Q = \sum_k Q^k_M \otimes Q^k_{M^c}$

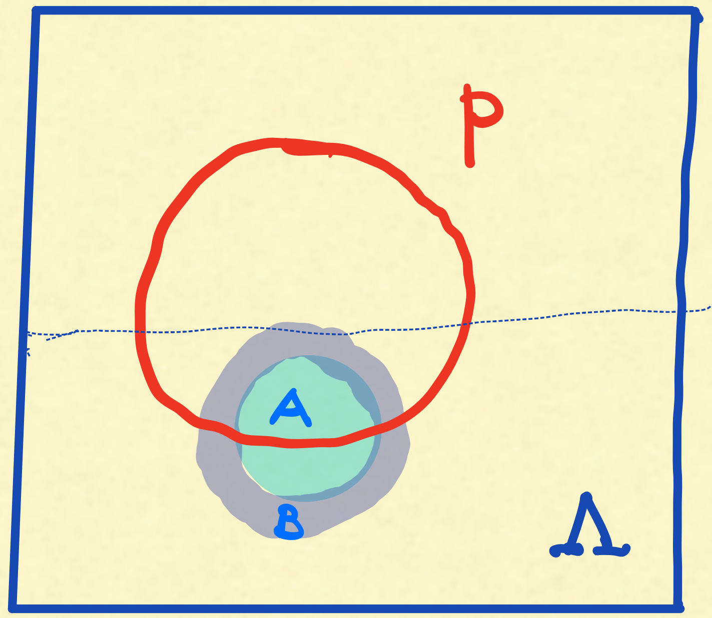
Example: Toric code stabilizers

$$S^X \infty S^Z = X_M Z_M \otimes Z_{M^c} X_{M^c} = -S^X S^Z$$

A witness for topological order

Local invisibility:

(motivated by Wilson loop operators)



P acts trivially on small enough regions A

i.e. $\rho_A = \mathrm{Tr}_{A^c}\left[P \rho_\Lambda P \right]$

Lemma (informal) [Haah (2013)]:

If locally invisible operators exist w.r.t $|\psi\rangle$ such that

Witness → $\langle \psi | P \infty Q | \psi \rangle \neq \langle \psi | P | \psi \rangle \langle \psi | Q | \psi \rangle$

then $|\psi\rangle$ can not be prepared with a constant depth circuit.

A witness for topological order

We define robust witness function ("twist correlator")

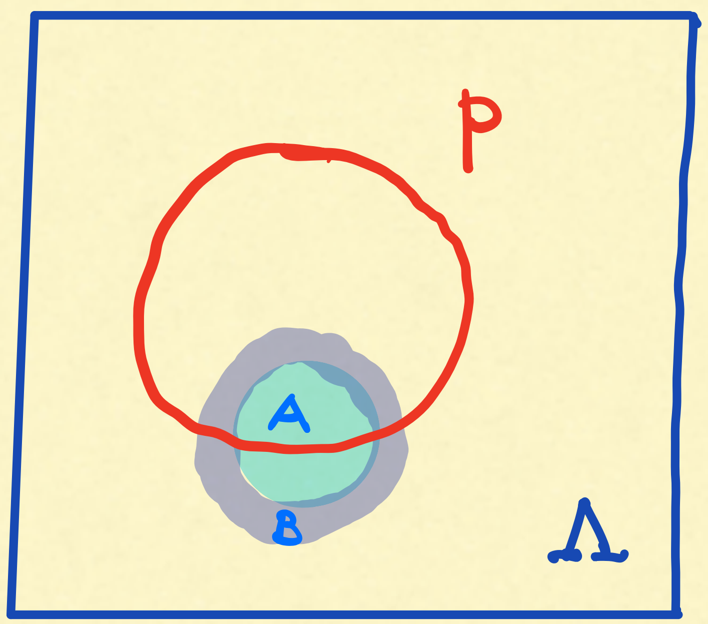
$$\boxed{C(P,Q)_{|\psi\rangle} := \left| \langle\psi| P \infty Q |\psi\rangle - \langle\psi|P|\psi\rangle\langle\psi|Q|\psi\rangle \right|}$$

↳ can use this if P and Q are only approximately locally invisible!

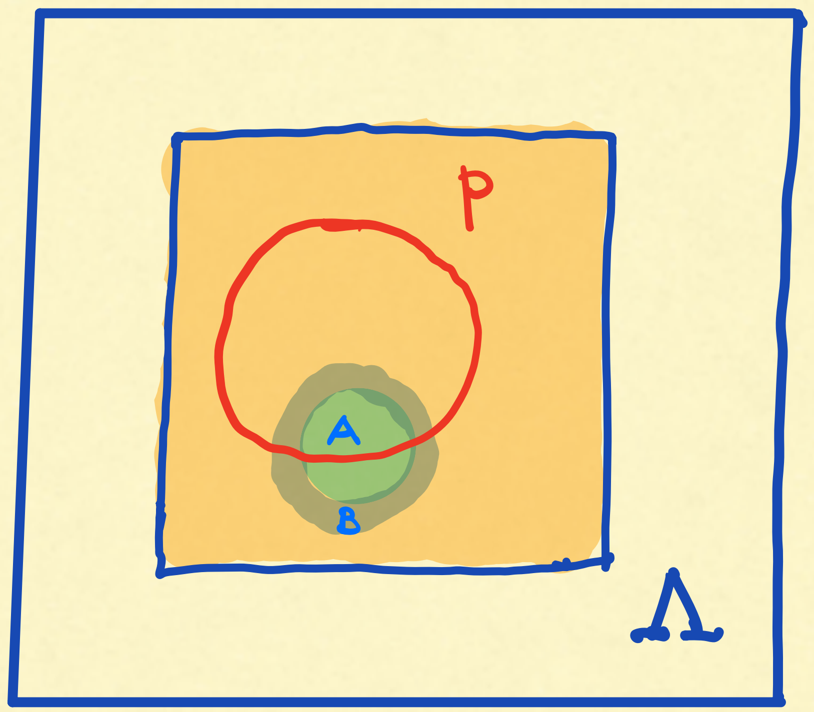
Approximate local invisibility:

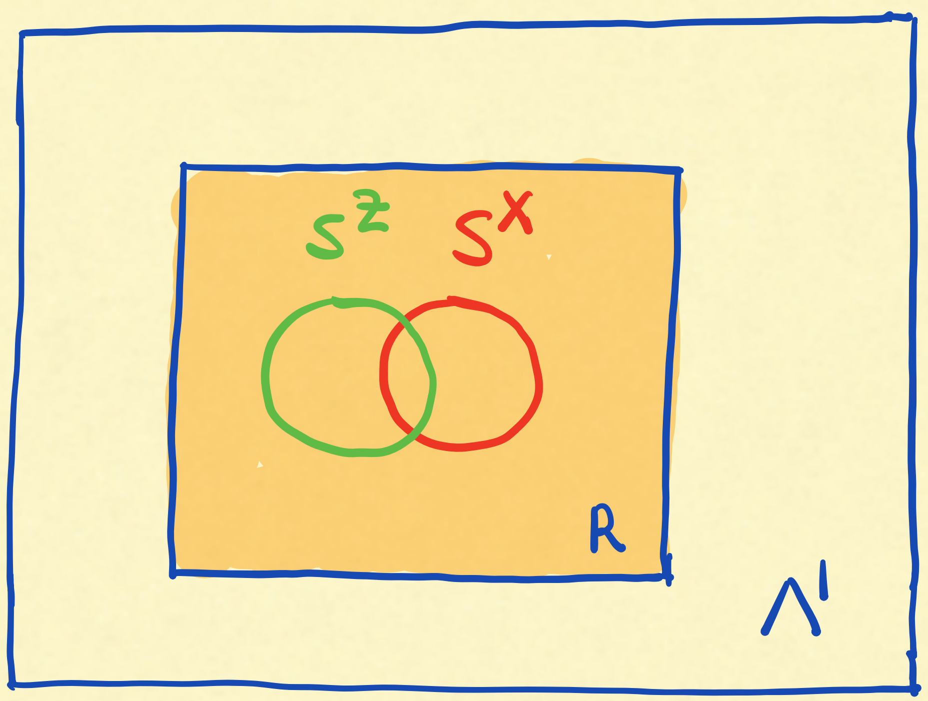
$$\boxed{\left\| \rho_A - \mathrm{Tr}_{A^c}[P\rho_\Lambda P] \right\|_1 \leq \delta}$$

P acts approximately like the identity on small enough patch A



Proving the bound





We show that:

Toric code stabilizers supported on ε-good subsystems are approximately locally invisible!

$$\boxed{\left\| \rho_A^\varepsilon - \mathrm{Tr}_{A^c}\left[S_R \, \rho_\Lambda^\varepsilon \, S_R \right] \right\|_1 \le 2\sqrt{|R|\varepsilon}}$$

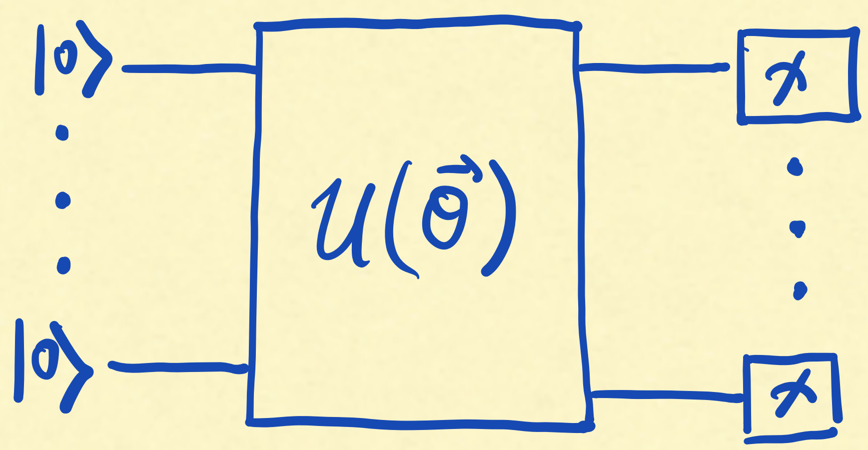
↪ Use our robust witness function on ρ_Λ^ε.

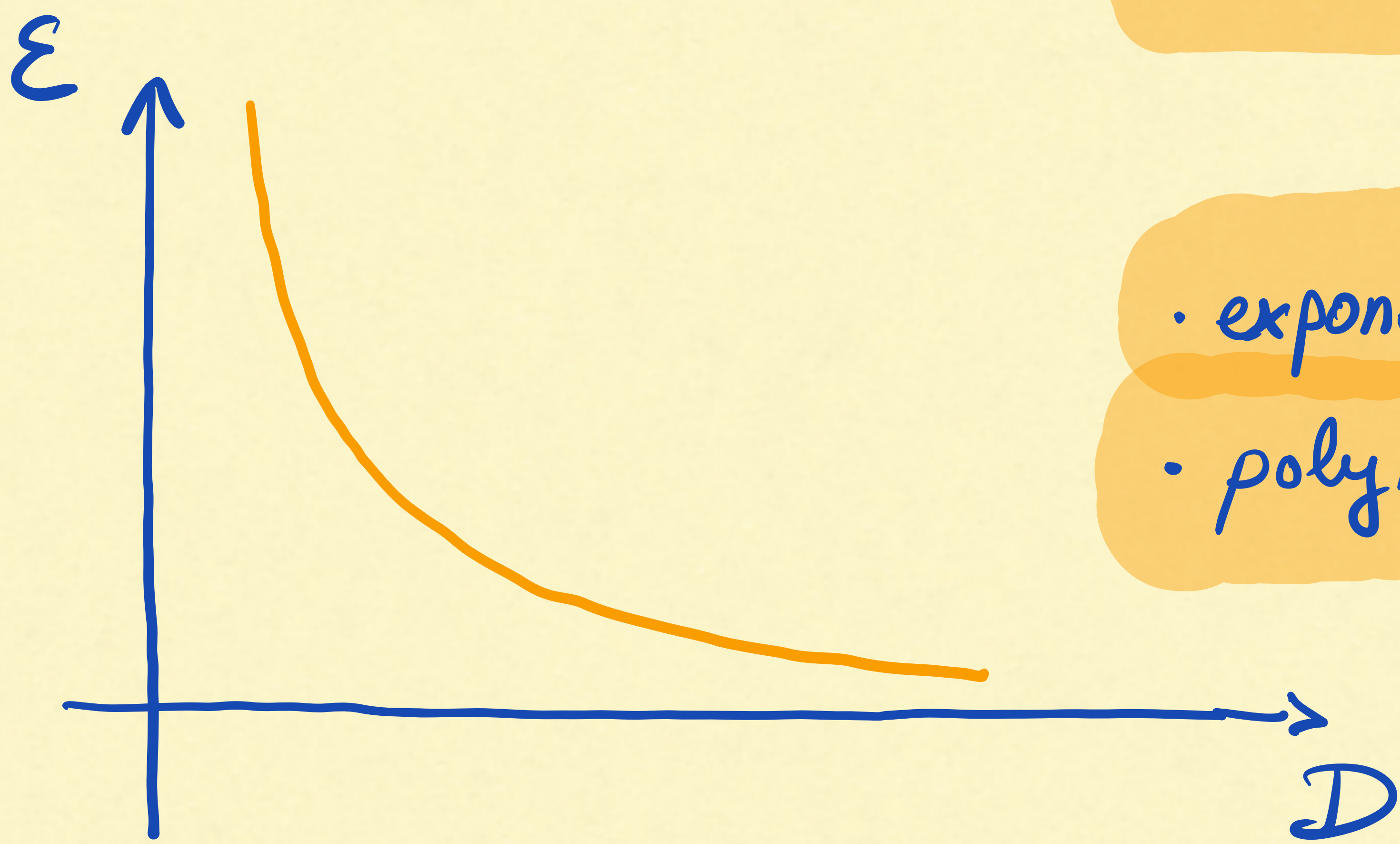
$$f(\varepsilon) \le \left| \langle \psi^\varepsilon | S^z \infty S^x | \psi^\varepsilon \rangle - \langle \psi^\varepsilon | S^z | \psi^\varepsilon \rangle \langle \psi^\varepsilon | S^x | \psi^\varepsilon \rangle \right| \le g(D)$$

Main result follows:

$$D = \Omega\left(\sqrt{1/\varepsilon}\right)$$

Summary and discussion





No-go result for preparing approximate groundstates in log-depth for topologically ordered models in 2D.

- exponential decay ✗
- polynomial decay ✓

Summary and discussion

Quantum Physics

[Submitted on 6 Oct 2022 (v1), last revised 8 Oct 2022 (this version, v2)]

NLTS Hamiltonians from classical LTCs

Zhiyang He, Chinmay Nirkhe

We provide a completely self-contained construction of a family of NLTS Hamiltonians [Freedman and Hastings, 2014] based on ideas from [Anshu, Breuckmann, and Nirkhe, 2022], [Cross, He, Natarajan, Szegedy, and Zhu, 2022] and [Eldar and Harrow, 2017]. Crucially, it does not require optimal-parameter quantum LDPC codes and can be built from simple classical LTCs such as the repetition code on an expander graph. Furthermore, it removes the constant-rate requirement from the construction of Anshu, Breuckmann, and Nirkhe.

Comments: This note is withdrawn due to an uncorrectable error. A detailed explanation on the withdrawal is hosted on my website at this https URL

On the technical side:

Proof technique for circuit depth lower bounds that is independent of groundspace degeneracy of Hamiltonian $\longrightarrow$ Generalize our formalism to lift assumptions on NLTS theorem?