Quantum correlations on the no-signaling boundary: self-testing and more

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Abstract

In device-independent quantum information, correlations between local measurement outcomes observed by spatially separated parties in a Bell test play a fundamental role. Even though it is long-known that the set of correlations allowed in quantum theory lies strictly between the Bell-local set and the no-signaling set, many questions concerning the geometry of the quantum set remain unanswered. Here, we revisit the problem of when the boundary of the quantum set coincides with the no-signaling set in the simplest Bell scenario. In particular, we prove that self-testing is possible in nontrivial classes of these common boundaries beyond the known examples of Hardy-type correlations (1). We also show that if the qubit strategies leading to an extremal nonlocal correlation are local-unitarily equivalent, a self-testing statement can be made based on this correlation. Interestingly, all these self-testing correlations on the no-signaling boundary found are provably non-exposed. An analogous characterization for the set of quantum correlations arising from finite-dimensional maximally entangled states is also provided.

Self-testing quantum components from Bell test and various sets of correlations

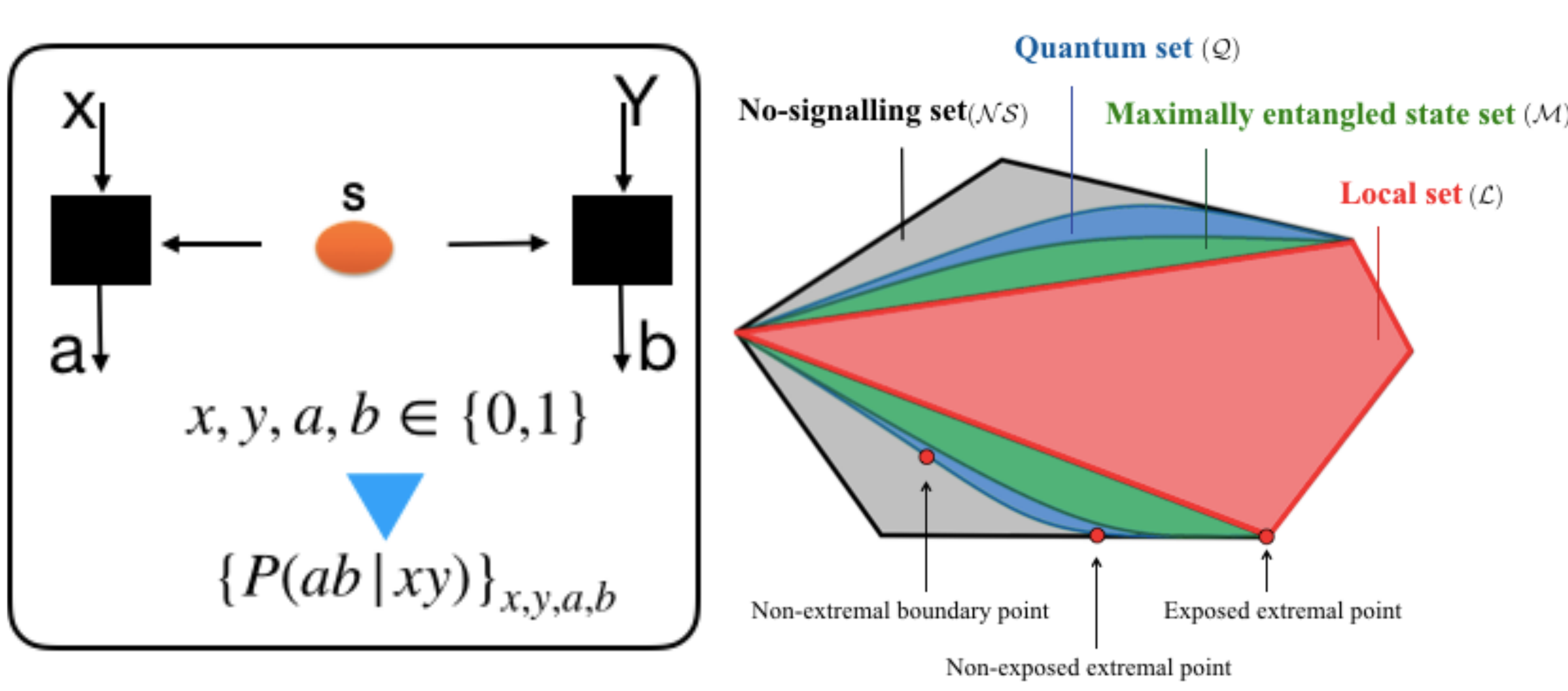

Figure 1: Certain correlations $\{P(ab|xy)\}_{xyab}$ allow self-testing specific state and measurements $\{|\psi\rangle, M^A_{a|x}, M^B_{b|y}\}$ up to some local operations. $\mathcal{L} \subset \mathcal{Q} \subset \mathcal{NS}$.

Results

Examples for a specific no-signaling face:

- Class 2b: $\{P(00|00) = P(11|10) = 0\}$
- $|\psi\rangle = \frac{1}{\sqrt{2}}(cos\alpha|01\rangle + |10\rangle - sin\alpha|11\rangle)$; $A_0 = \sigma_z, B_0 = -\sigma_z, A_1 = cos2\alpha\ \sigma_z - sin2\alpha\ \sigma_x, B_1 = cos2\beta\ \sigma_z - sin2\beta\ \sigma_x$. $\alpha = \pi/6, \beta = \pi/4$.
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$$\vec{P}_{\mathrm{Q,2b}} := \begin{array}{cc|cc|cc} & & \multicolumn{2}{c|}{x=0} & \multicolumn{2}{c}{x=1} \\ & & 0 & 1 & 0 & 1 \\ \hline y=0 & 0 & 0 & \frac{1}{2} & \frac{1}{8} & \frac{3}{8} \\ & 1 & \frac{3}{8} & \frac{1}{8} & \frac{1}{2} & 0 \\ \hline y=1 & 0 & \frac{3}{16} & \frac{9}{16} & \frac{9}{16} & \frac{3}{16} \\ & 1 & \frac{3}{16} & \frac{1}{16} & \frac{1}{16} & \frac{3}{16} \end{array} \qquad (1)$$

Robustness of self-testing:

- $S_{CHSH} = \sum_{a,b,x,y}(-1)^{xy\oplus x\oplus y}P(ab|xy)$.
- $\rho_{swap} = Tr_{AB}[\Phi(\rho_{AB} \otimes |00\rangle\langle 00|)\Phi^\dagger]$; $\mathcal{F} = \langle\psi|\rho_{swap}|\psi\rangle$ (3).

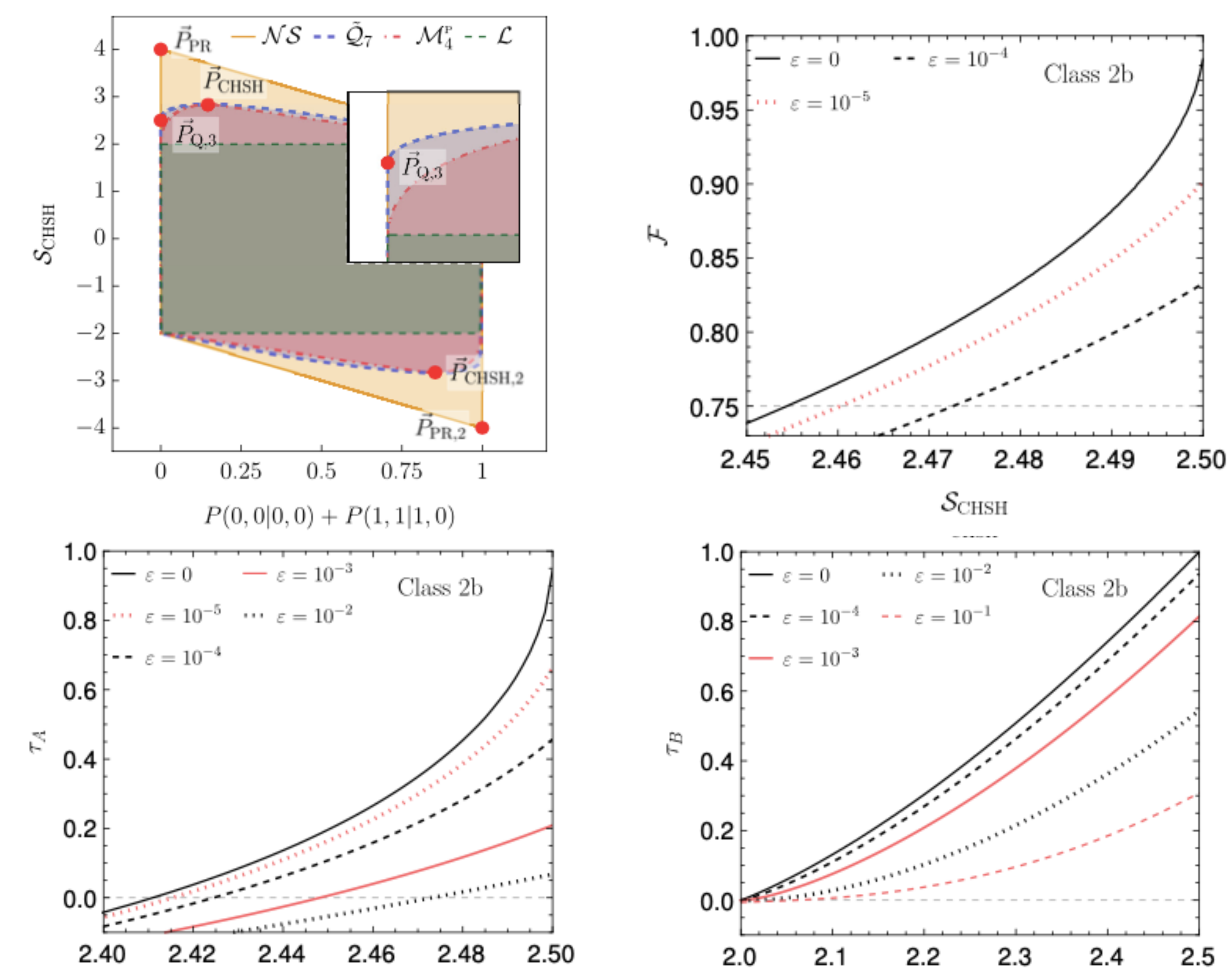

Figure 2: In the 1st figure of the above panel, different sets of correlations are shown on a projection on a particular face. Right top figure shows robustness of self-testing state while bottom two figures show that of Alice and Bob's measurement.

Class	Necessarily $\in \mathcal{L}$?	$\mathcal{L}^c \cap \mathcal{M} \neq \emptyset$?	Self-test?	Non-Exposed?	$\mathcal{S}^{\max}_{\mathrm{CHSH}}$	$E(\psi)$
4b	Yes	N/A	No	No	2	1
3a	No	No	Yes (2)	Yes (4)	2.3607	0.6742
3b	No	No	Yes	Yes	2.2698	0.7748
2a	No	Yes	Yes	Yes	2.5	1
2b	No	No	Yes	Yes	2.5	0.8113
2c	No	No	Yes (5)	Yes	2.4312	0.7794
1	No	Yes	Yes	Yes	2.6353	0.9255

Figure 3: Summary of our results are given in the table. $\mathcal{NS}$ boundaries are specified by the number of zeros in the correlation. Examples of quantum correlations on $\mathcal{NS}$ are found and also maximal value of constrained Bell-CHSH inequality obtained together with the entropy of entanglement of the underlying state. All the extreme quantum points on $\mathcal{NS}$ boundaries are non-exposed. For details of the above discussions about motivations and results see (6).

References

[1] L. Hardy, Phys. Rev. Lett. **71**, 1665 (1993).
[2] R. Rabelo et al. Phys. Rev. Lett. **109**, 180401 (2012).
[3] H. Yang et al., Phys. Rev. Lett. **113**, 040401 (2014).
[4] K. T. Goh et al., Phys. Rev. A **97**, 022104 (2018).
[5] A. Rai et ai., Phys. Rev. A **103**, 062219 (2021).
[6] K. S. Chen et al., arXiv:2207.13850 (2022). .