Interplay of nonlocality and Incompatibility breaking qubit channels

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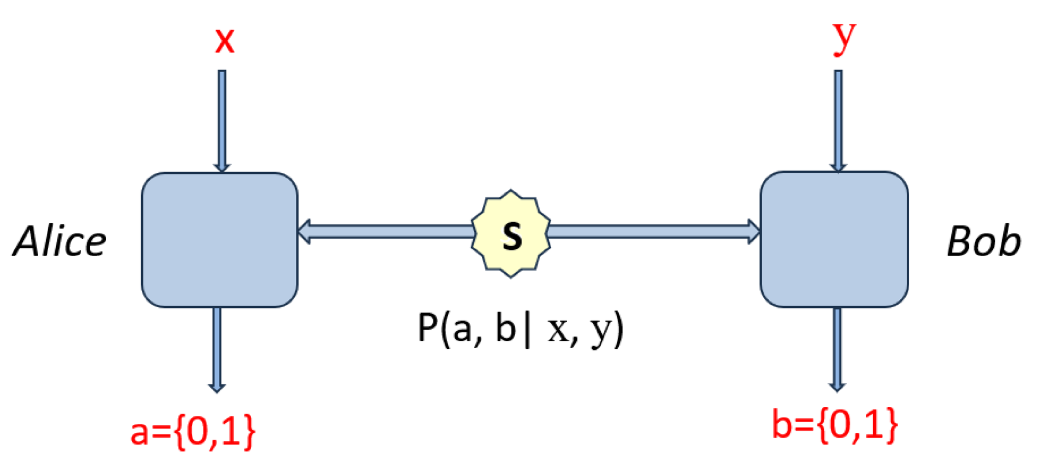
Abstract

Nonlocality [1,2] and incompatibility [3] are not only of foundational interest but also act as important resources for quantum information theory. Here, we investigate these notions in the context of qubit channels. In Bell CHSH scenario, we prove that if the conjugate of a channel is incompatibility breaking, then the channel is itself nonlocality-breaking and the converse also holds provided the channel is unital. We investigate this relation in tripartite scenario by considering some well- known states like GHZ and W states and using the notion of Mermin and Svetlichny nonlocality. Further, we identify the set of unital qubit channels that is Mermin/Svetlichny nonlocality breaking irrespective of the input state.

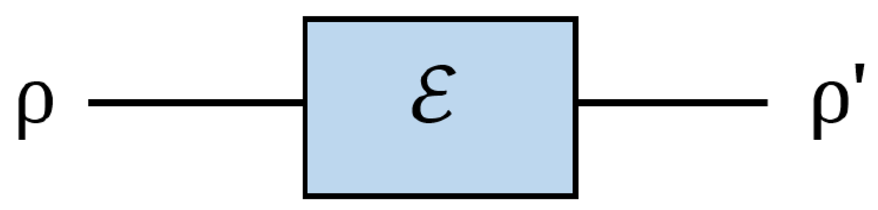
Nonlocality in the Bell Scenario

Alice and Bob, two spatially seperated observers perform measurements on, say x and y on their sites respectively and collect the output statistics P(a, b|x, y)[1,2].



Quantum Channel

Quantum Channel: A quantum channel ($\mathcal{E} : \mathcal{L}(\mathcal{H}^{\mathcal{A}}) \rightarrow \mathcal{L}(\mathcal{H}^{\mathcal{B}})$) is a CPTP map which takes input as state and outputs another state.



In the Kraus operator formalism, $$\rho' = \mathcal{E}(\rho) = \sum_{j=1}^{n} K_j \rho K_j^{\dagger}$$

where, $$\sum_j K_j^{\dagger} K_j = \mathbf{I}$$

Incompatibility breaking channel (IBC):

Definition 1: Any quantum channel is said to be incompatibility breaking channel if the outputs are compatible with any choice of input observables [4].

Nonlocality breaking channel (NBC):

Definition 2: A nonlocality breaking channel can be defined as a channel which when applied to a system leads to a state which is local [5].

Maximum violation of the Mermin/Svetlitchny Inequality

Definition 3: For any three-qubit quantum state ρ , the maximum value of the Mermin and the Svetlitchny operator is bounded as [6,7],

max|$< M >_\rho$ | <=$2\sqrt{2}\lambda_1$ max|$< S >_\rho$ | <=$4\sqrt{2}\lambda_1$

where $< M >_\rho$=Tr [Mρ] and λ_1 is the maximum singular value of the matrix M =($M_{j,ik}$), with M =($M_{i,jk}$) , with M=($M_{i,jk}$)=Tr[Mρ].

Results

1. Equivalence of CHSH nonlocality breaking and incompatibility breaking channels

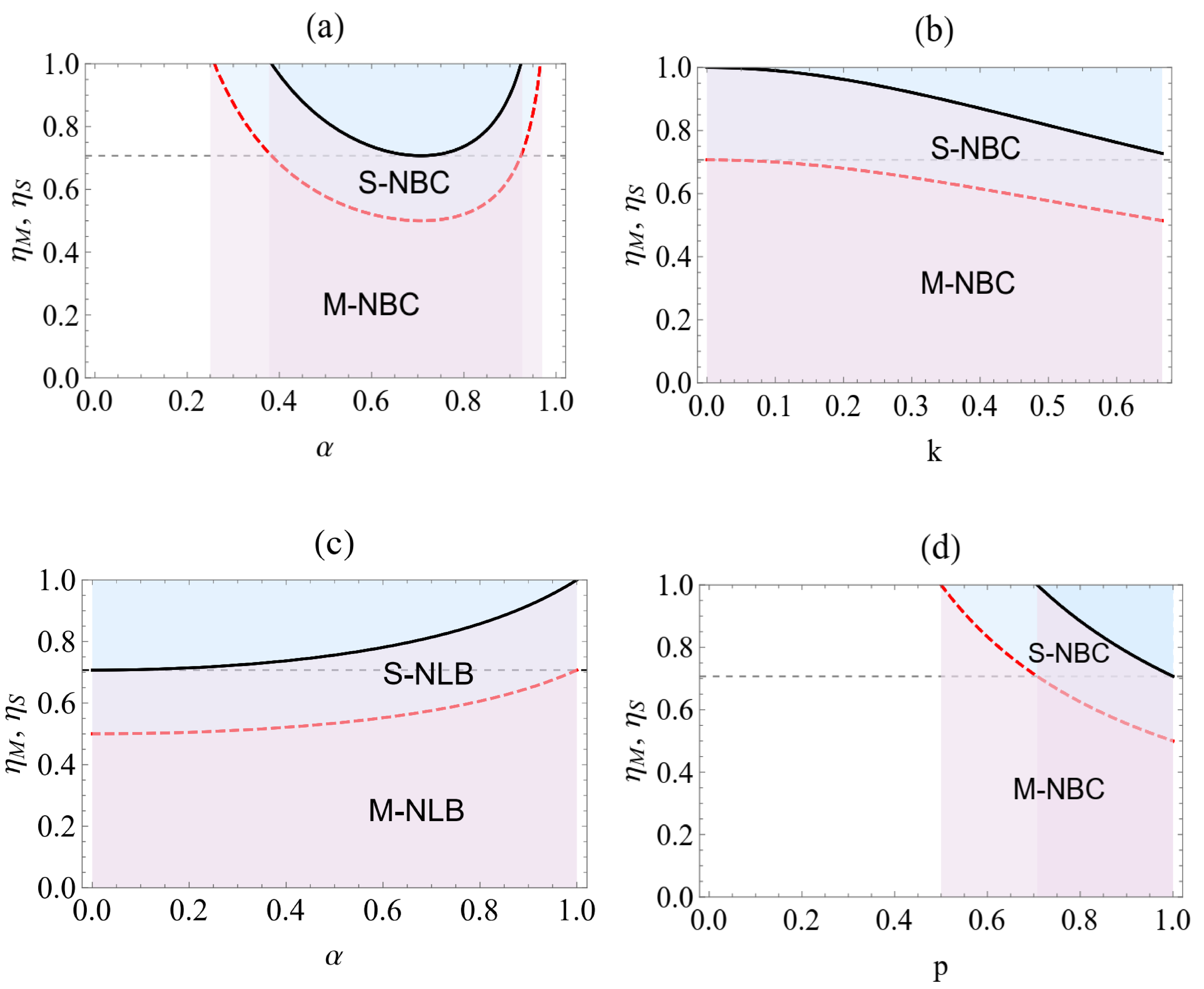
Theorem 1. If the conjugate of a qubit channel is 2-IBC, then channel itself is CHSH-NBC.

Theorem 2. If a qubit channel is CHSH-NBC, then its conjugate is 2-IBC.

2. Genuine tripartite nonlocality breaking channel

Definition 4: For any three-qubit state ρ_{ABC} , a given qubit channel is said to be absolute or genuine nonlocality breaking if acting on any qubit, it gives a state, which satisfies the Mermin inequality or Svetlichny inequality.

3. NBC and IBC in the tripartite scenario



The region below the dashed(red) and solid(black) curve in (a), (b), (c) and (d) corresponds to M-NBC and S-NBC, plotted against the (dimensionless) state coefficients. The pairwise incompatibility condition pertains too all points below the horizontal dashed line.

Conclusion

- We show an equivalency relation between CHSH nonlocality and incompatibility breaking channel.
- We extend this study in tripartite scenario using Mermin/Svetlichny inequality studied in some well-known states.
- We found that within certain range of channel and state parameter incompatibility assures nonlocality for those state.

References

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