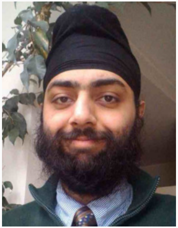
On the complexity of hybrid quantum computation

Andru Gheorghiu (Chalmers University of Technology)



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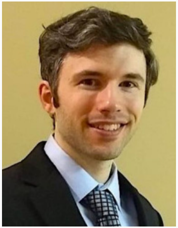
(Caltech)

Andrea Coladangelo

(University of Washington)



Matt Coudron

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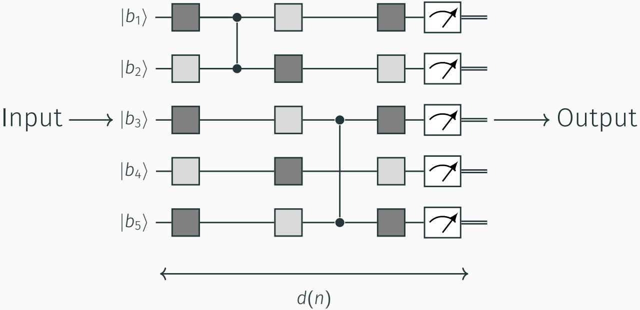
Hendrik Waldner

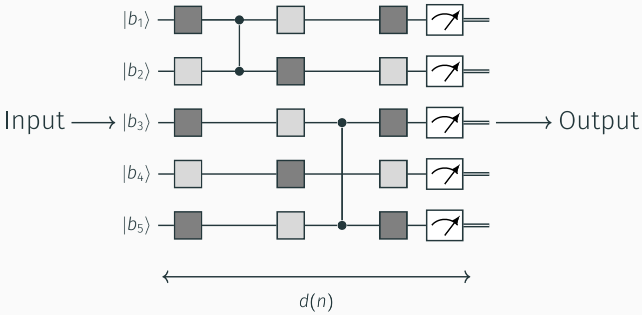
(Max Planck Institute, University of Maryland)

Hybrid quantum-classical computation



Quantum d depth.

Hybrid quantum-classical computation



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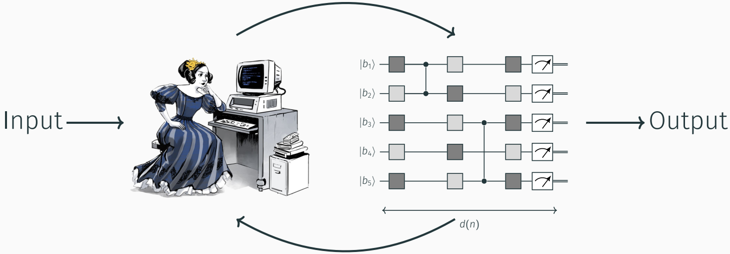

Classical invokes Quantum d depth.

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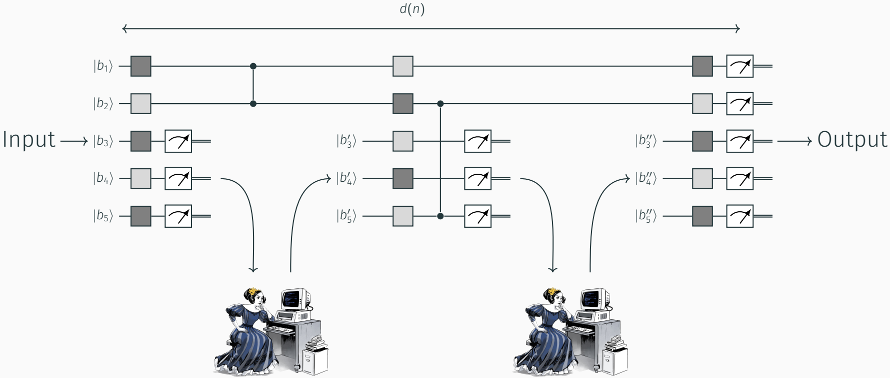

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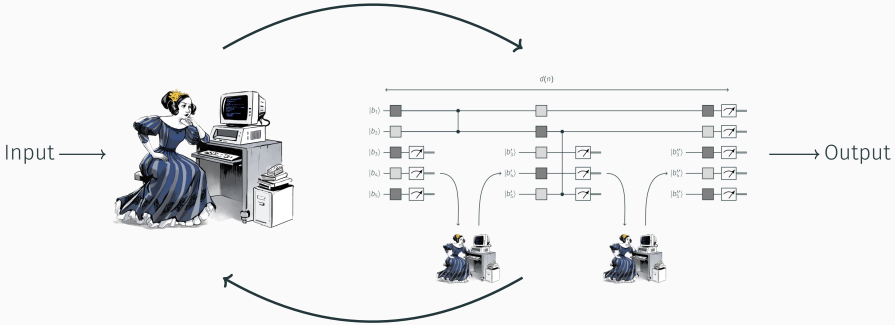

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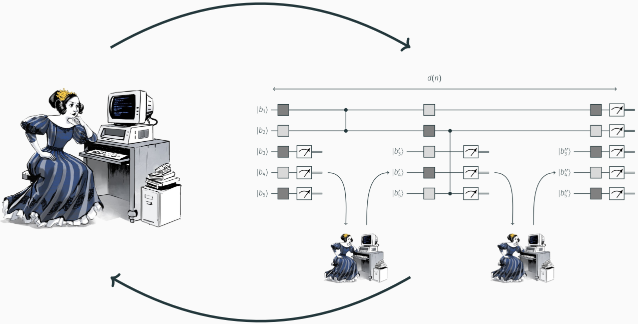

$\longrightarrow$ Output

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Q_d, CQ_d, QC_d, CQC_d.

QNC, $\mathsf{BPP}^{\mathsf{QNC}}$, $\mathsf{QNC}^{\mathsf{BPP}}$, $\mathsf{BPP}^{\mathsf{QNC}^{\mathsf{BPP}}}$

(when $d(n) = \log^{O(1)}(n) \equiv \mathrm{polylog}(n)$)

Hybrid quantum-classical computation

$$
\begin{tikzcd}
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Jozsa's conjecture[4] (reformulated)

$$\mathsf{CQC}_{\mathrm{polylog}} = \mathsf{BQP}.$$

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[4] [Jozsa, 2005]

Limitations of hybrid models

Aaronson's challenge[5]

Does there exist an *oracle* relative to which Jozsa's conjecture is false? Can the oracle then be *instantiated?*

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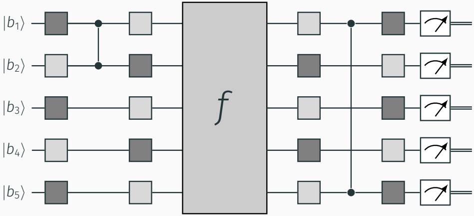

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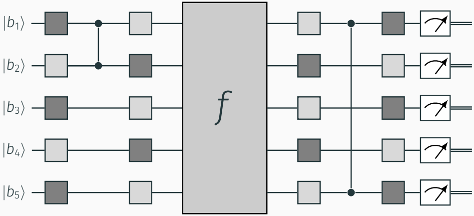

$$\mathsf{Q}_d^f, \mathsf{CQ}_d^f, \mathsf{QC}_d^f, \mathsf{CQC}_d^f, \mathsf{BPP}^f, \mathsf{BQP}^f.$$

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Relativized quantum depth separation[6]

There exists a function, f, such that $\mathsf{CQ}^f_{\mathrm{polylog}} \cup \mathsf{QC}^f_{\mathrm{polylog}} \subsetneq \mathsf{BQP}^f$.

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- No known instantiations.
- Any instantiation should have at most $O(\mathrm{polylog}(n))$ depth.

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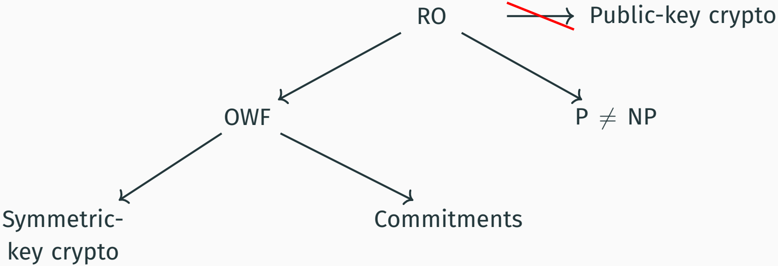

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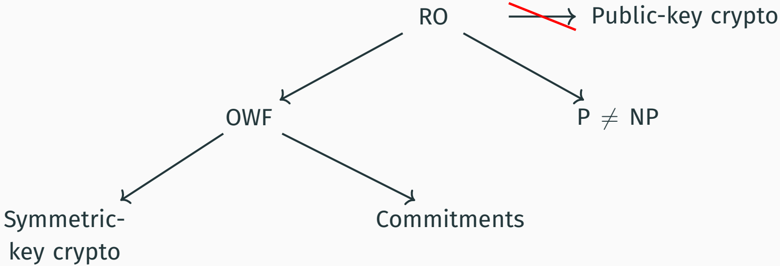

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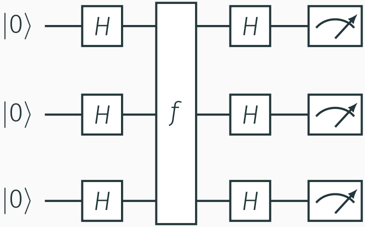

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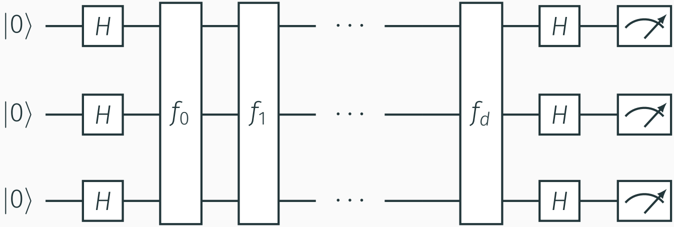

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Claim: New problem is not in CQC_d^f.

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That problem is also in NP^f (efficiently verifiable).

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We obtain a *proof of d quantum depth protocol*.[10]

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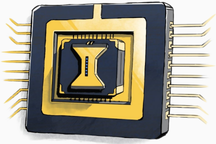

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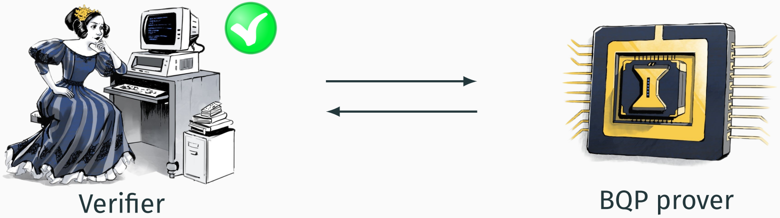

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There is a **BQP** prover that makes the verifier accept whp.

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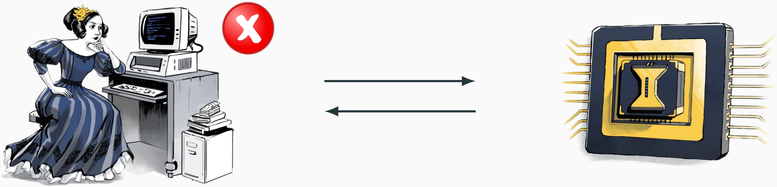

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CQC_d prover

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No $\mathbf{CQC}_d$ prover can make the verifier accept whp (in the QROM).

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Thanks!