The Learnability of Pauli noise

Senrui Chen

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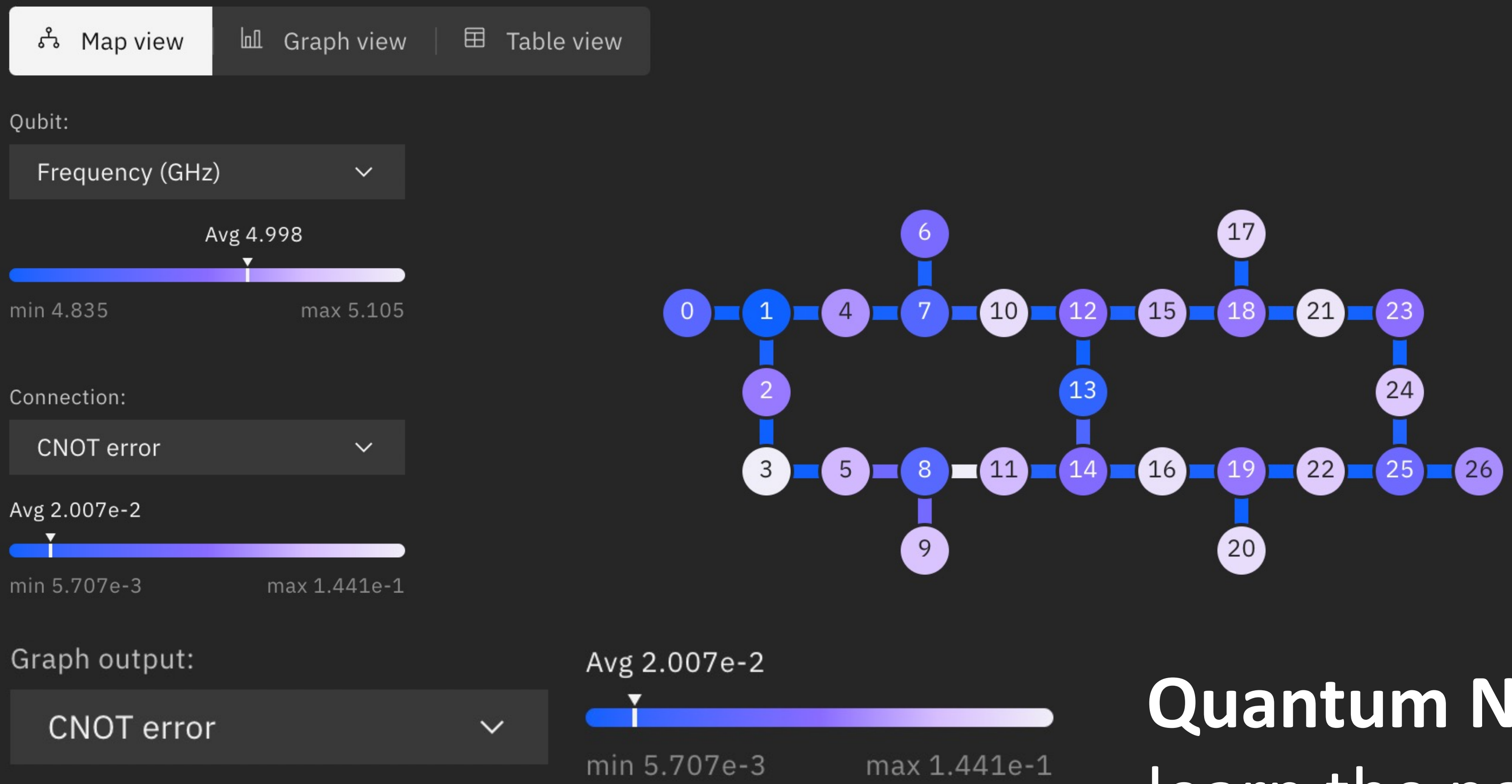
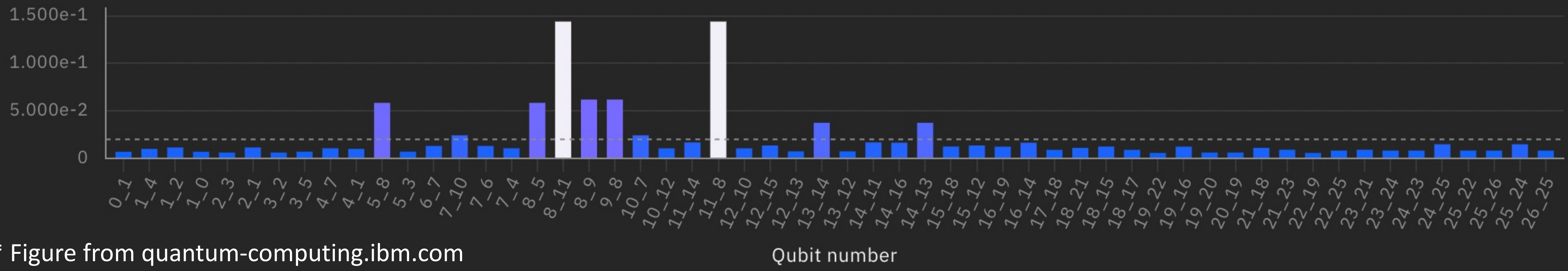
Based on *Nature Communications* ***14***, *52 (2023)*
with *Yunchao Liu*, Matthew Otten, Alireza Seif, Bill Fefferman, and Liang Jiang*

THE UNIVERSITY OF CHICAGO



Noise: The challenge for practical quantum computing systems

Quantum Noise Characterization: learn the noise in quantum devices

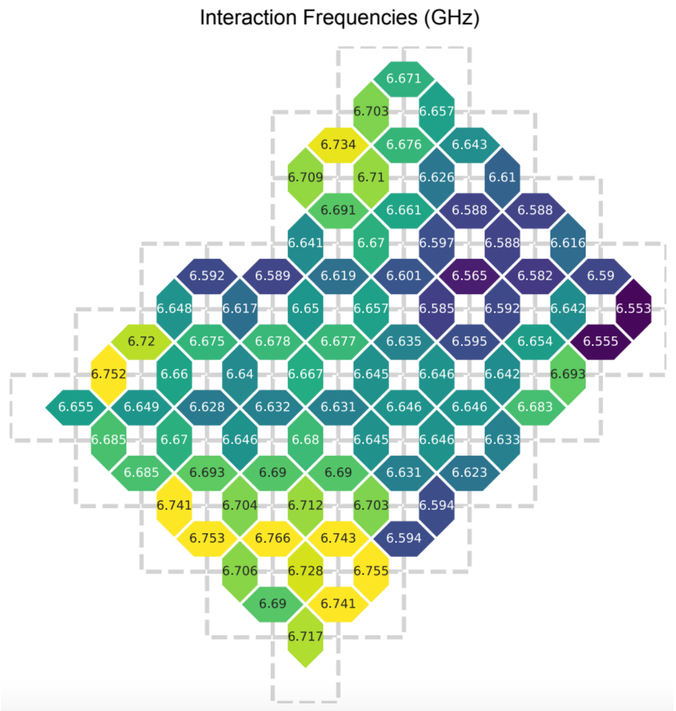


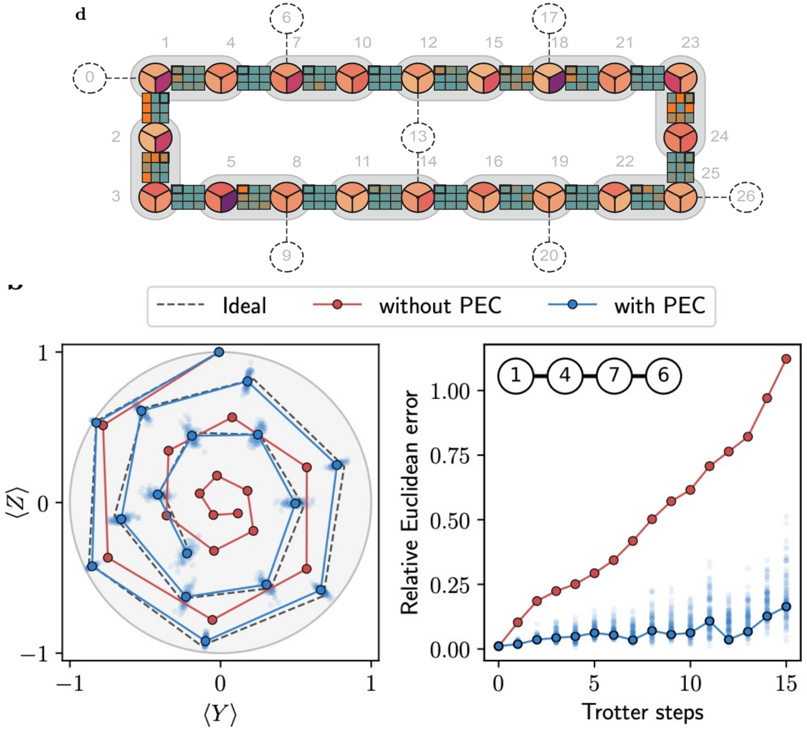
* Figure from quantum-computing.ibm.com

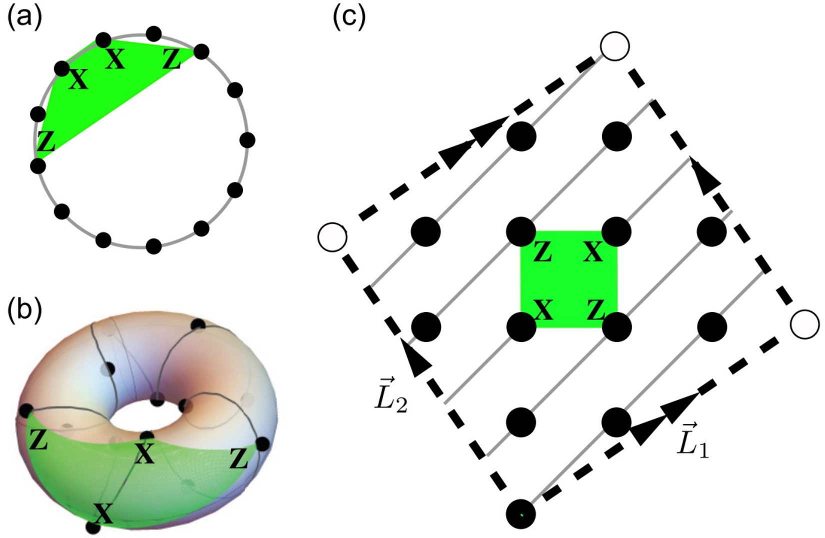
Why noise characterization?

Understanding and improving hardware

Effective error mitigation and correction schemes







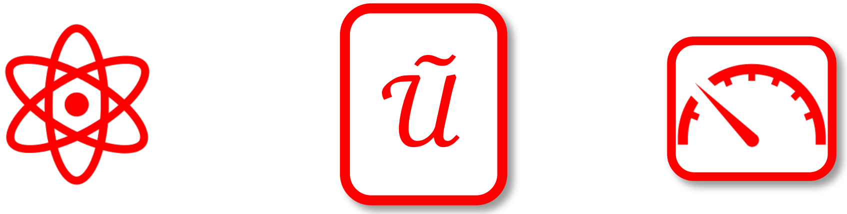
Google, Nature 2019

E Berg et al., IBM, arXiv 2201.0986

Q Xu et al., Phys. Rev. Research 5, 013035

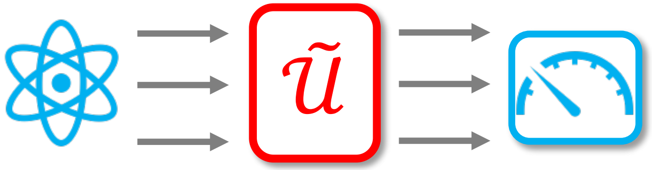
Learning the noise

- Circuit-level noise characterization:

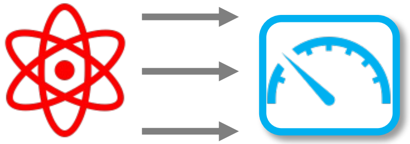


State preparation (SP) | Gate | Measurement (M)

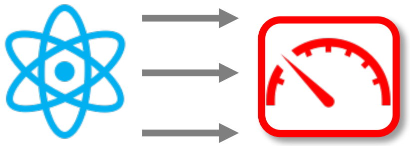
- Problems for tomography-based method …



Process tomography



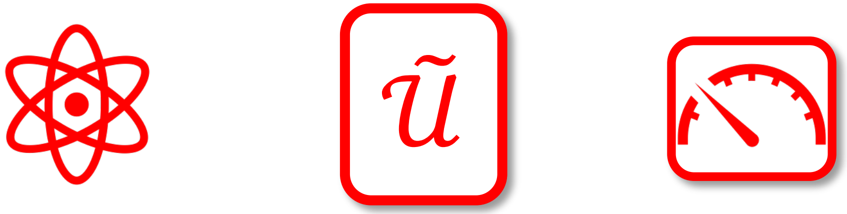
State tomography



Detector tomography

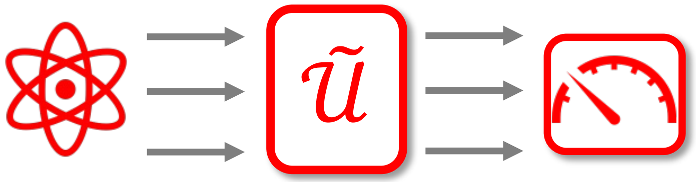
Learning the noise

- Circuit-level noise characterization:



State preparation (SP) Gate Measurement (M)

- Problems for tomography-based method …



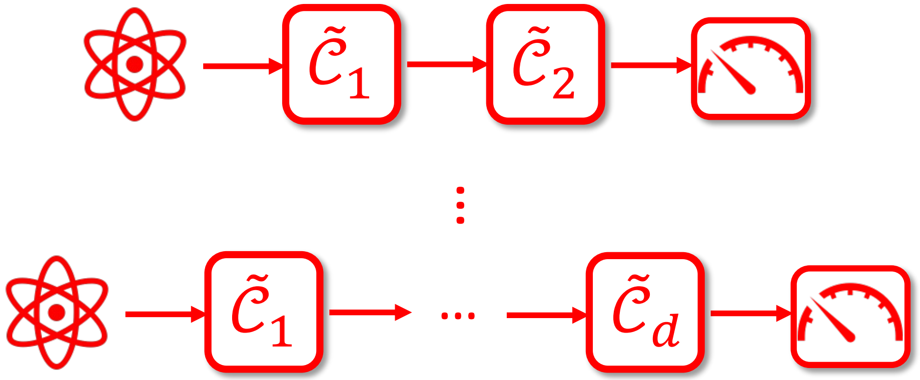
ibm_washington, 22-03-08

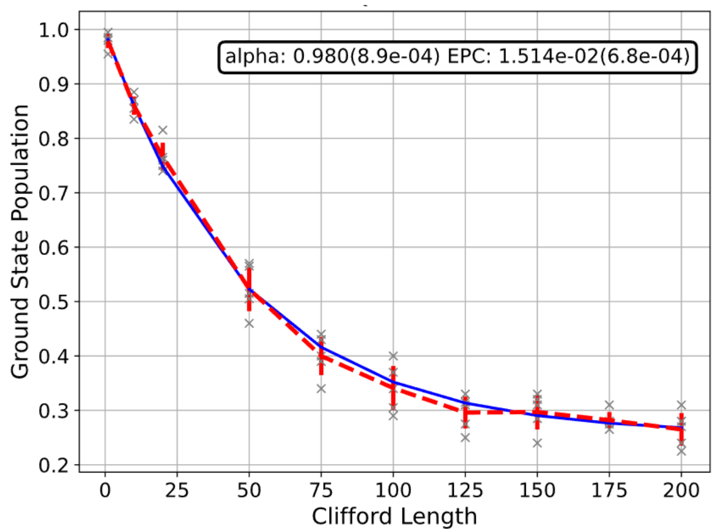
Median CNOT Error:	1.235e-2
Median Readout Error:	1.470e-2

  - All are noisy! SPAM noise can be non-negligible.

- Question: what (if anything) can be reliably learned about the noise?

Randomized benchmarking (RB)

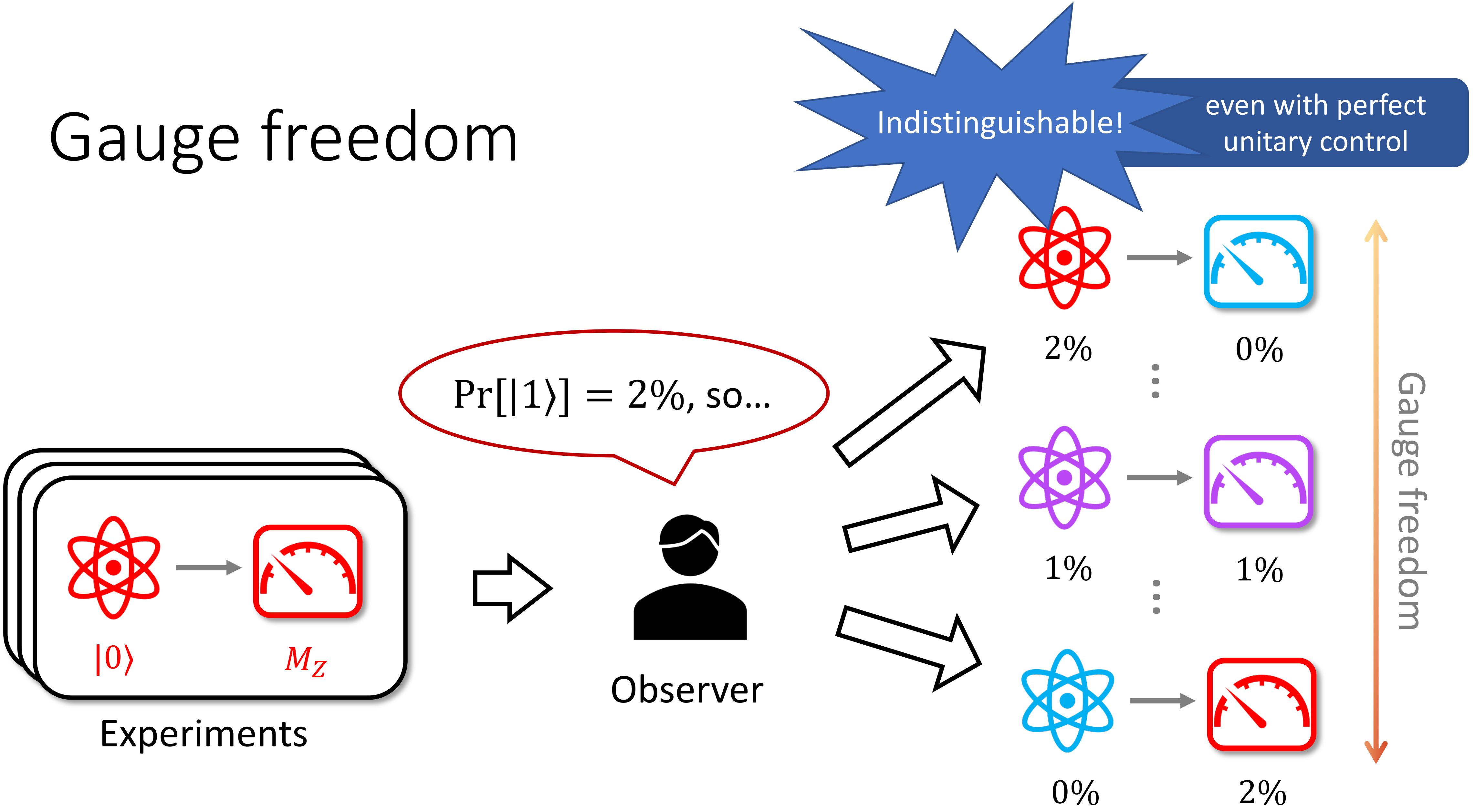




Randomized Benchmarking, Qiskit Textbook

- Separate **average gate fidelity** from **SPAM noise** .
- Need strong assumptions: **gate-independent** noise for Clifford
  - Difficulty arises with gate-dependent noise

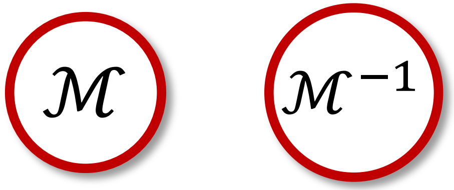
Gauge freedom

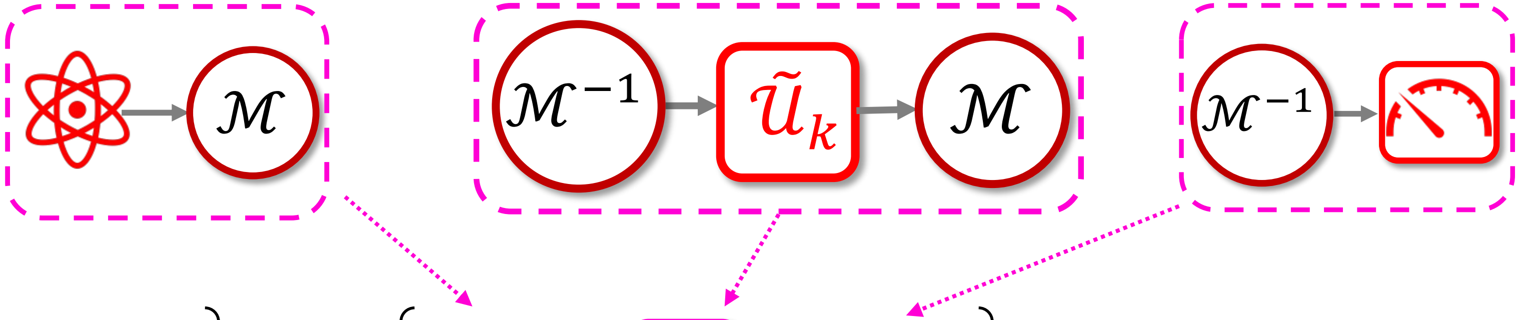


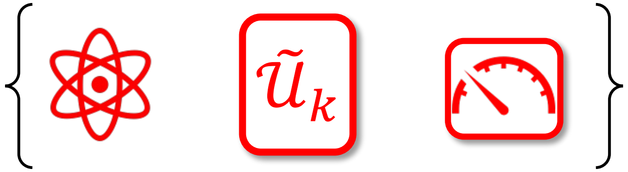
*Assume depolarizing noise (for simplicity)

Gauge transform

- Noisy gateset (SPAM+gate): 

- Gauge transform map (invertible): 

- Gauge transform: 

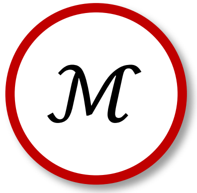
- Claim: 

and 

are **indistinguishable**

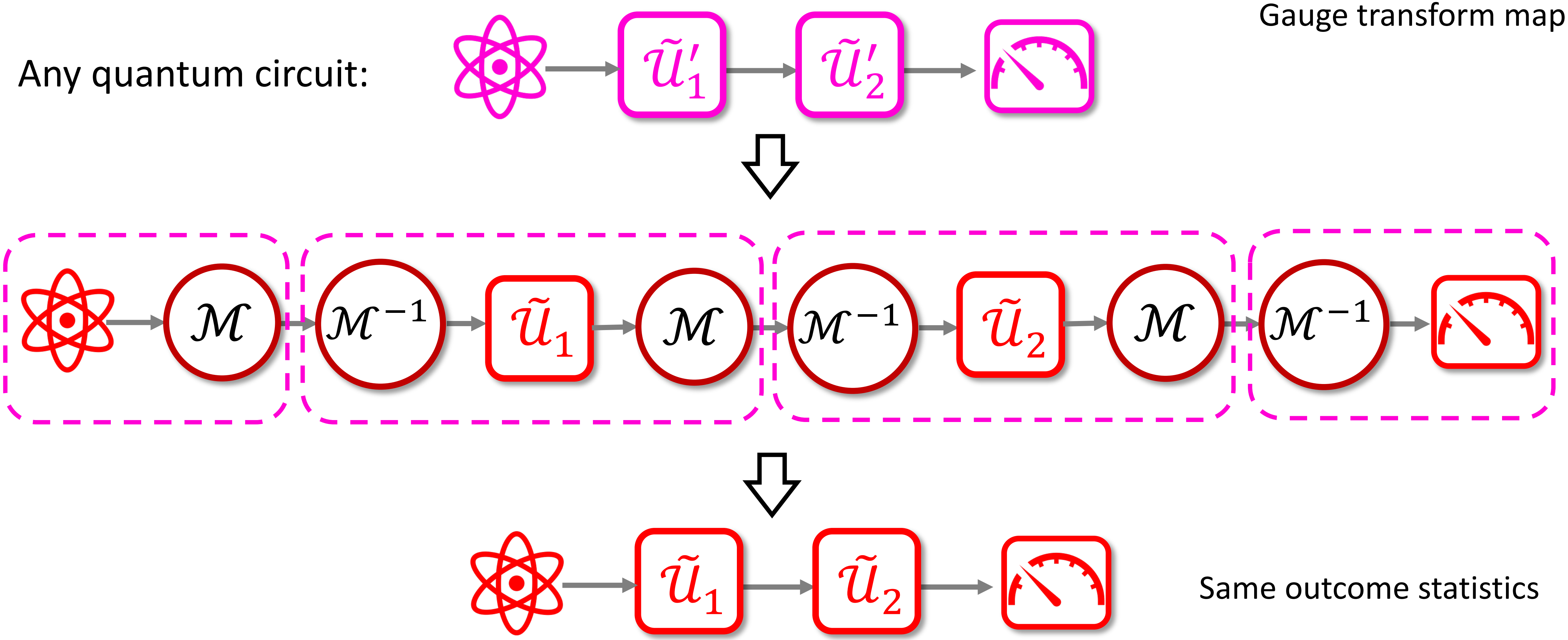
See, e.g., “Gate set tomography”. E Nielsen et al., Quantum 5, 557 (2021).

Proof of indistinguishability



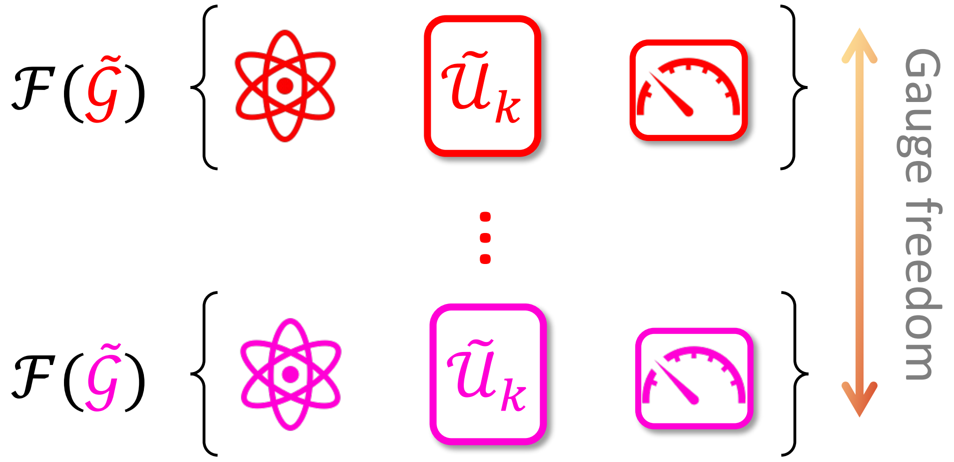
Gauge transform map

Any quantum circuit:



Same outcome statistics

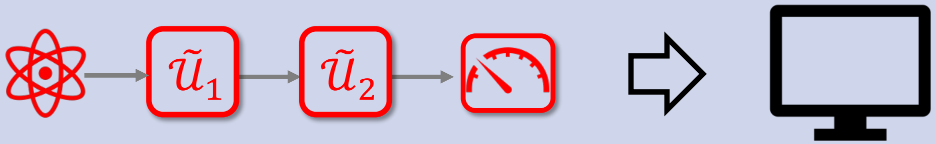
Gauge and noise learnability



- Gauge transform relates **indistinguishable** noisy gateset
  - Needs to preserve physicality (i.e., CPTP)
- **Learnable** functions: invariant under any gauge transform
  - E.g., Idle tomography.
- **Unlearnable** functions can only be determined up to **gauge freedom**
  - (though they have deterministic underlying values!)

Why care about noise learnability?

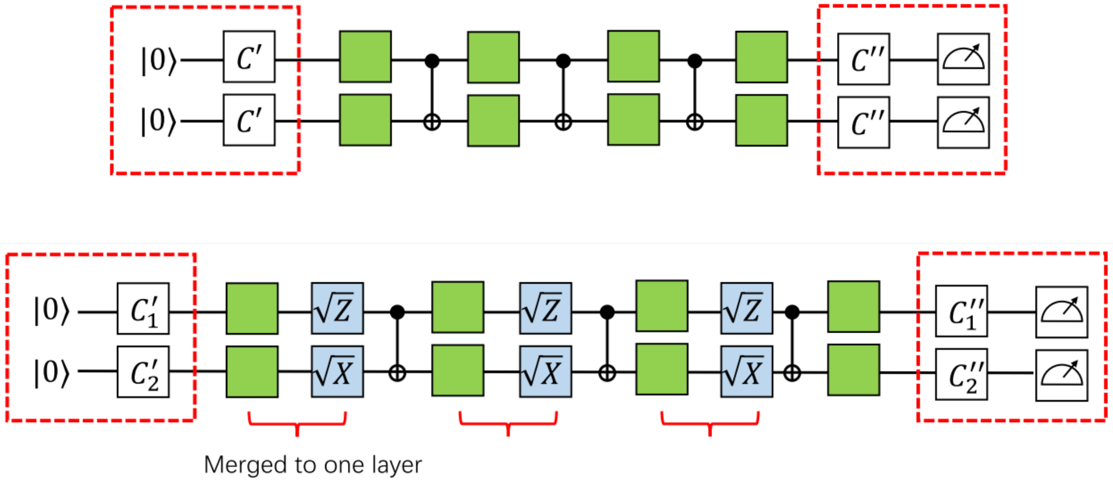
- Recall the two applications of noise characterization

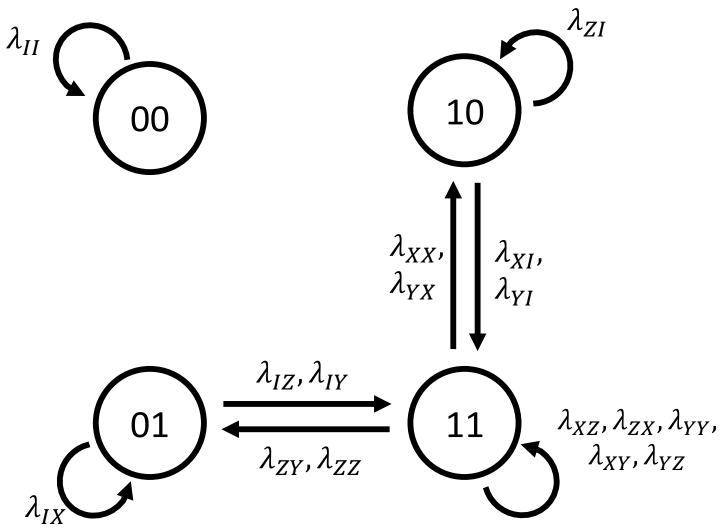
Hardware improvement	Software improvement
Unlearnability prevent accurate identification of noise source  (E.g., improving initialization or measurements?)	Important to know all learnable degrees of freedom for, e.g., accurate error mitigation. 

See also *"Foundations for learning from noisy quantum experiments"*. *Huang, Flammia, Preskill. arXiv 2204.13691*

Summary of results

- We precisely characterize the learnable degrees of freedom for **Pauli noise model**
- We give explicit protocol to estimate all learnable information of noise
- Experimental results highlight the practical significancy of noise unlearnability.





Pauli channel

- Pauli channel - two equivalent definitions:

$$\Lambda(\rho) = \sum_{a \in \mathrm{P}^{\mathrm{n}}} p_a P_a \rho P_a = \sum_{b \in \mathrm{P}^{\mathrm{n}}} \lambda_b P_b \, \mathrm{Tr}[\rho P_b]/2^n$$

  - $P_a \in \mathrm{P}^{\mathrm{n}} \equiv \{I, X, Y, Z\}^{\otimes n}$ – n-qubit Pauli operator.
  - $\{p_a\}_a$ – Pauli error rates. $\{\lambda_b\}_b$– Pauli fidelities.

Shrinkage rate on axis P_b

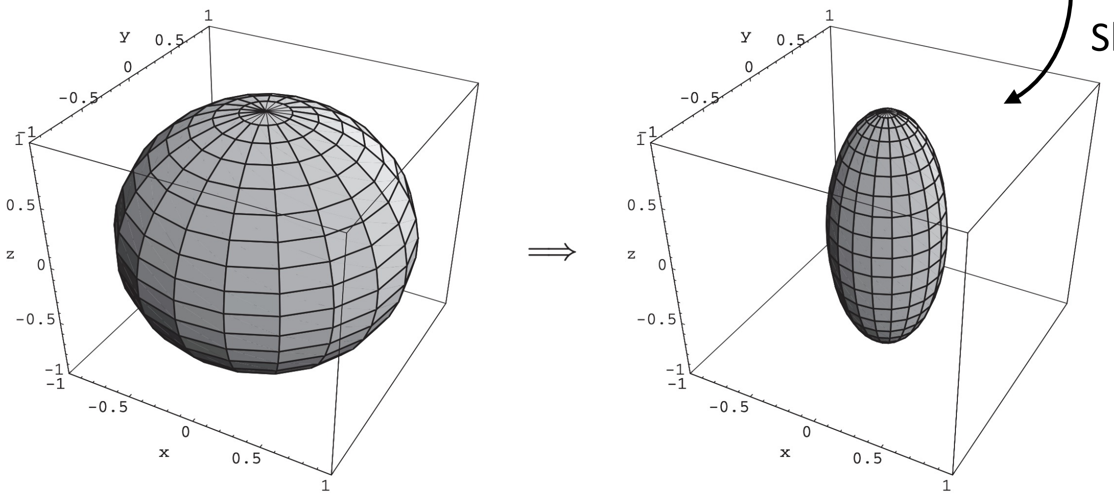


Fig. 8.9, Nielsen & Chuang

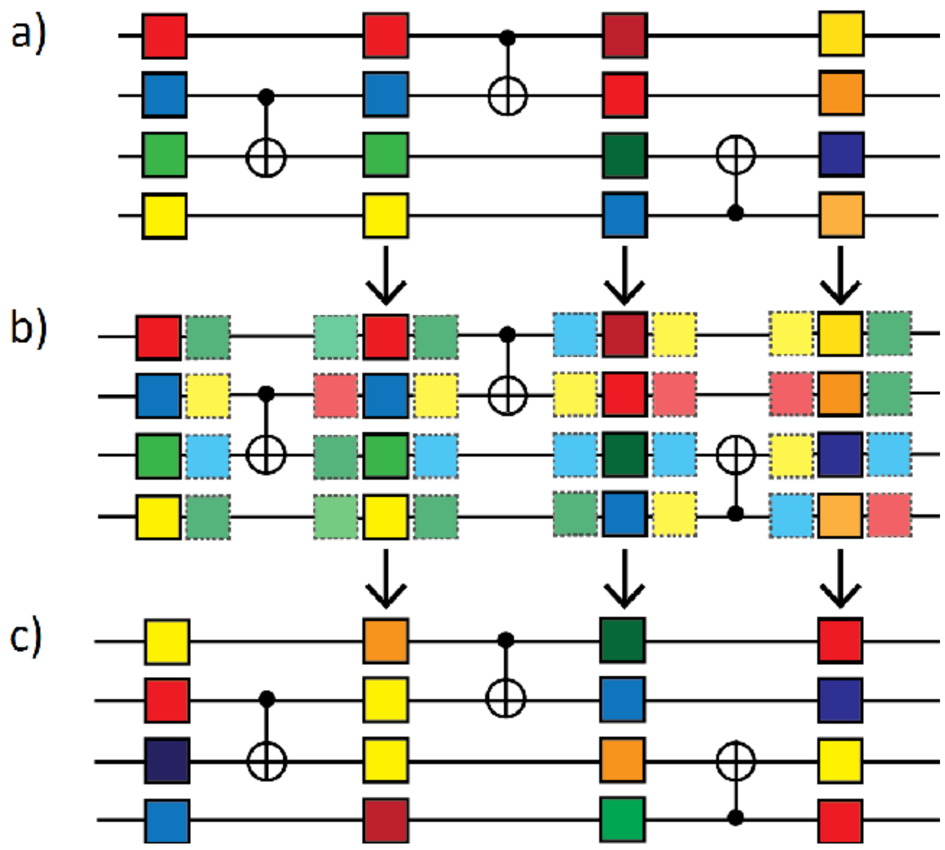
	I	X	Y	Z
p	0.7	0	0	0.3
λ	1	0.4	0.4	1

E.g., Phase-flip noise channel

Pauli noise model

- Noise assumption (n-qubit system):
  1) **Single-qubit unitary**: noiseless
  2) **Multi-qubit Clifford**: n-qubit gate-dependent Pauli noise $\tilde{\mathcal{G}} = \mathcal{G} \circ \Lambda_{\mathcal{G}}$.
  3) **SPAM**: unknown Pauli noise
  4) **Noise not too large**: $\lambda_a > 0, \forall a$

- Standard for randomized compiling
  - $\Lambda_{\mathcal{G}}$ can be interpreted as dressed noise of a cycle of SQ+MQ gates.

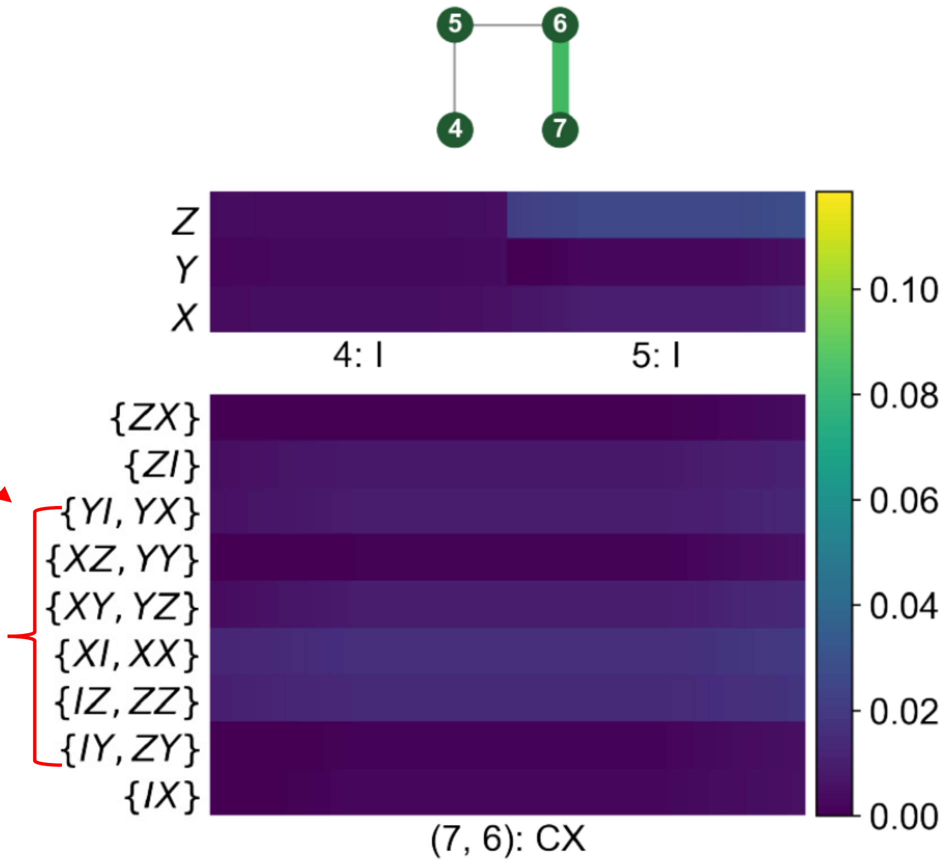


J Wallman, J Emerson, Phys. Rev. A **94**, 052325 (2016)

Gauge and Learnability of Pauli noise

- Question: Is $\Lambda_{\mathcal{G}}$ **learnable** (SPAM-robustly)?
- State-of-the-art: Cycle benchmarking (CB) characterizes $\Lambda_{\mathcal{G}}$ up to some **degeneracy**

  [A Erhard et al., Nat. Comm., 10, 5347]
- We gives a complete characterization on the **learnability** of Pauli noise, and construct an **explicit protocol** to learn everything learnable.



A Hashim et al.,
Phys. Rev. X **11**, 041039 (2021)

Main result

- **Thm 1 (Individual learnability)**.

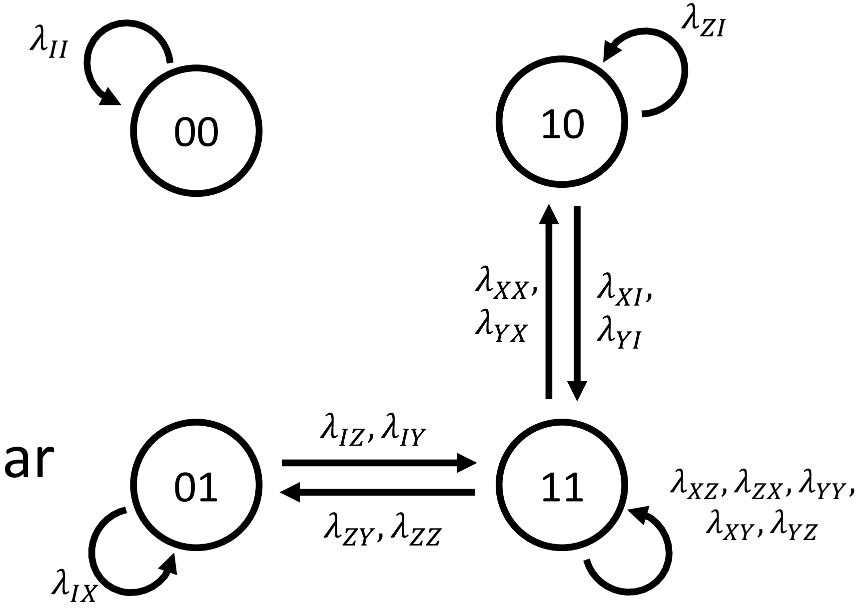
Given a Clifford $\mathcal{G}$ and a Pauli P_a, the Pauli fidelity $\lambda_a^{\mathcal{G}}$ is **learnable** iff $\mathcal{G}$ does not change the **pattern** of P_a.

  - Pattern: $P \mapsto \{0,1\}^n$ via $\{X, Y, Z\} \mapsto 1, I \mapsto 0$
  - E.g. for CNOT: $\lambda_{IX}, \lambda_{YY}$ is learnable; $\lambda_{XX}, \lambda_{XI}$ is unlearnable.

- **Thm 2 (Learnable degrees of freedom)**.

Define **log Pauli fidelity** $l_a^{\mathcal{G}} \coloneqq \log \lambda_a^{\mathcal{G}}$ for all $\mathcal{G}, P_a$. Any linear functions f of $\{l_a^{\mathcal{G}}\}_{\mathcal{G},a}$ is learnable iff f lives in the **cycle space** of the pattern transform graph for given gateset

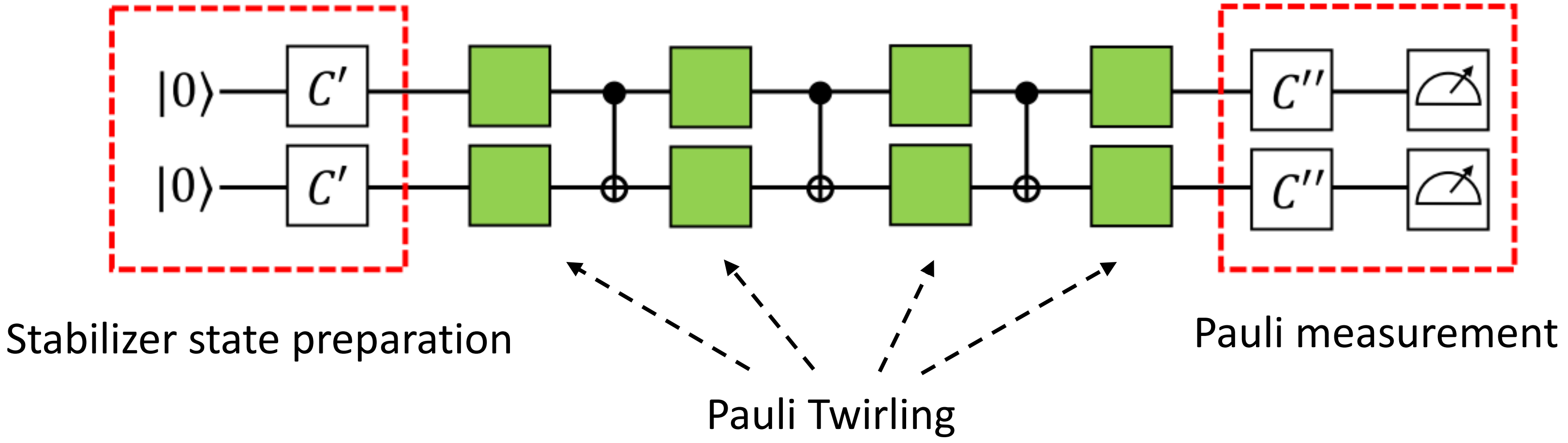
  - E.g. of CNOT: $\sqrt{\lambda_{XX}\lambda_{XI}}$ is learnable
  - 1st-order approximation for any functions, e.g., $\{p_a\}_a$



Pattern transform graph for CNOT

Proof Sketch – Learnable part

1. For $\mathcal{G}(P_a) = P_a$, cycle benchmarking (CB) gives **SPAM-robust** estimate for $\lambda_a^{\mathcal{G}}$



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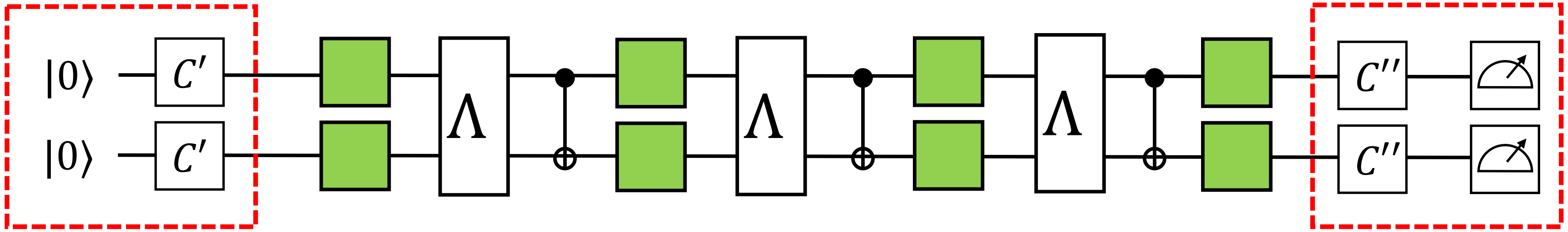

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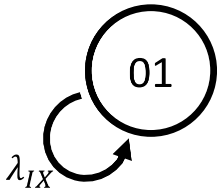
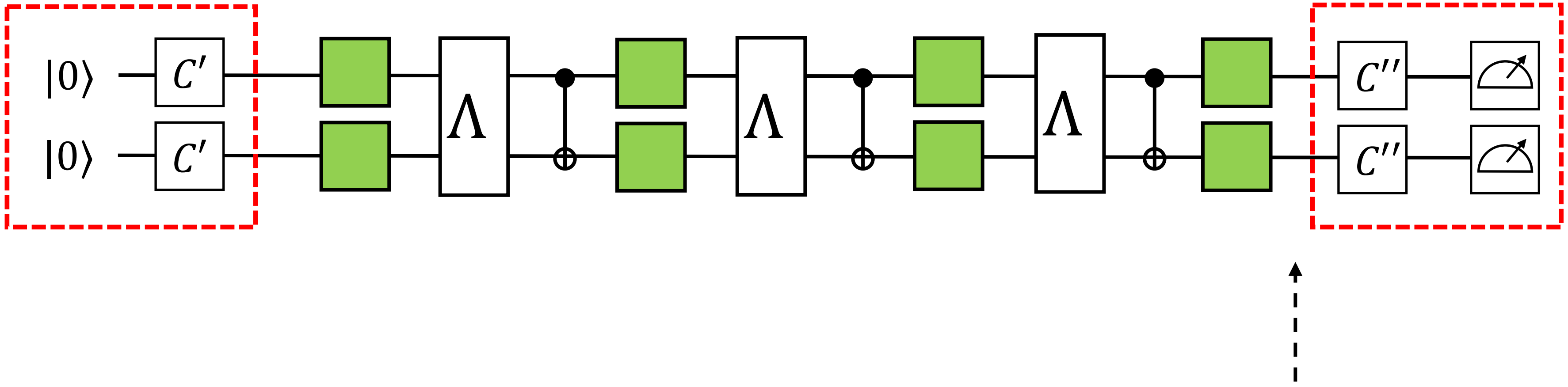


$\mathrm{CNOT}(IX) = IX$

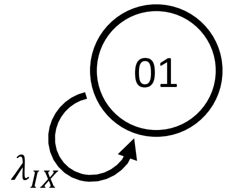


$\widetilde{IX} = \xi_{\mathrm{M},IX}\, IX$

$\xi_{\mathrm{M},IX}$: Pauli Fidelity for measurement noise

Proof Sketch – Learnable part

1. For $\mathcal{G}(P_a) = P_a$, CB gives **SPAM-robust** estimate for $\lambda_a^{\mathcal{G}}$



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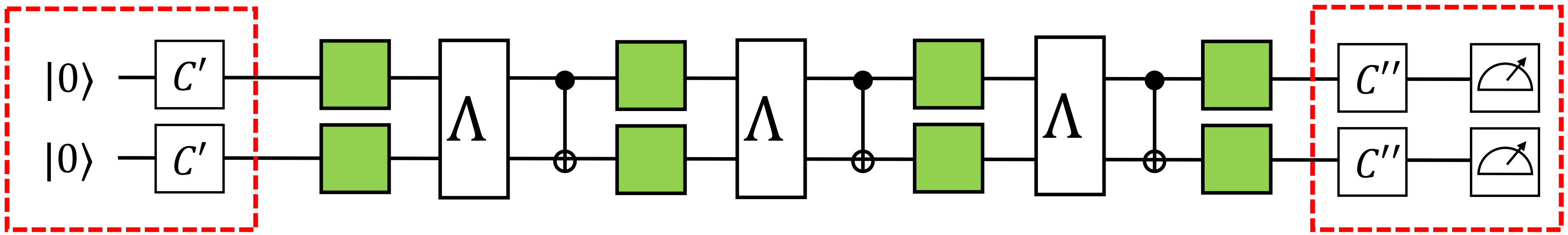

$$P\ \widetilde{IX}\ P = \pm\xi_{\mathrm{M},IX} IX$$

The $\pm$ can be easily corrected, thus omitted

$\xi_{\mathrm{M},IX}$: Pauli Fidelity for measurement noise

Proof Sketch – Learnable part

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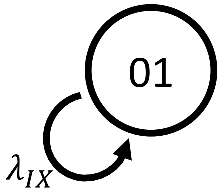
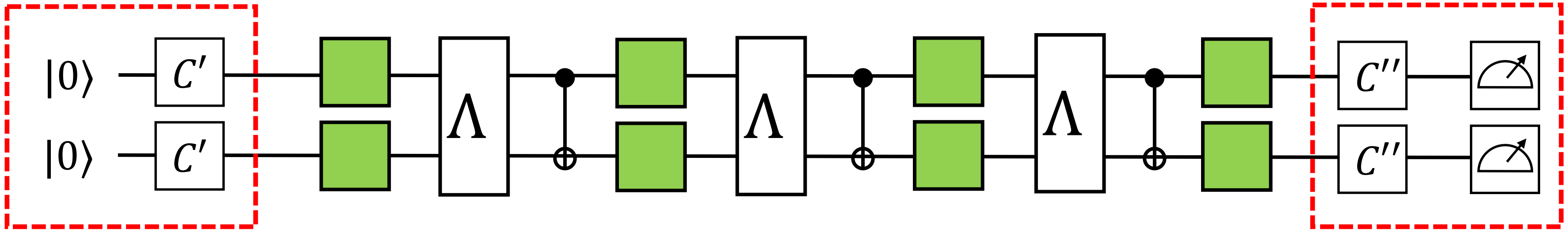


$\mathrm{CNOT}(IX) = IX$

$\widetilde{\mathrm{CNOT}} = \mathrm{CNOT} \circ \Lambda$

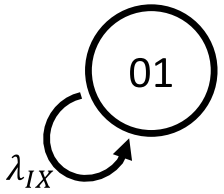


$\widetilde{\mathrm{CNOT}}^{\dagger}(\widetilde{IX}) = \lambda_{IX}\xi_{\mathrm{M},IX} IX$

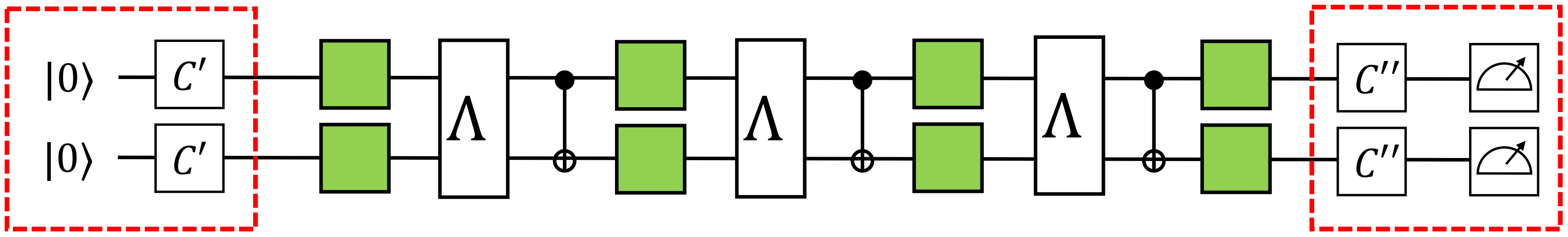
$\xi_{\mathrm{M},IX}$: Pauli Fidelity for measurement noise

Proof Sketch – Learnable part

1. For $\mathcal{G}(P_a) = P_a$, CB gives **SPAM-robust** estimate for $\lambda_a^{\mathcal{G}}$



$\mathrm{CNOT}(IX) = IX$



$\lambda_{IX}^2 \xi_{\mathrm{M},IX} IX$

$\xi_{\mathrm{M},IX}$: Pauli Fidelity for measurement noise

Proof Sketch – Learnable part

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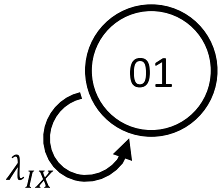

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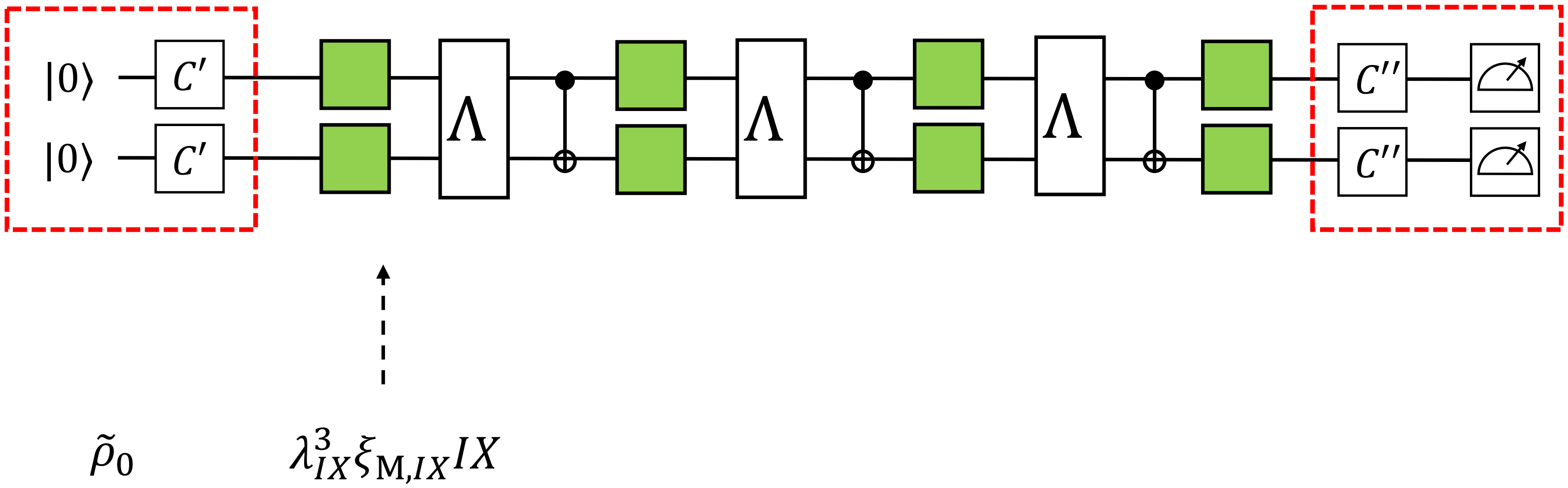

$\xi_{\mathrm{M},IX}$: Pauli Fidelity for measurement noise

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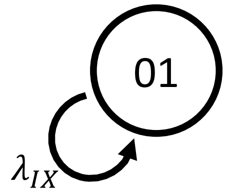

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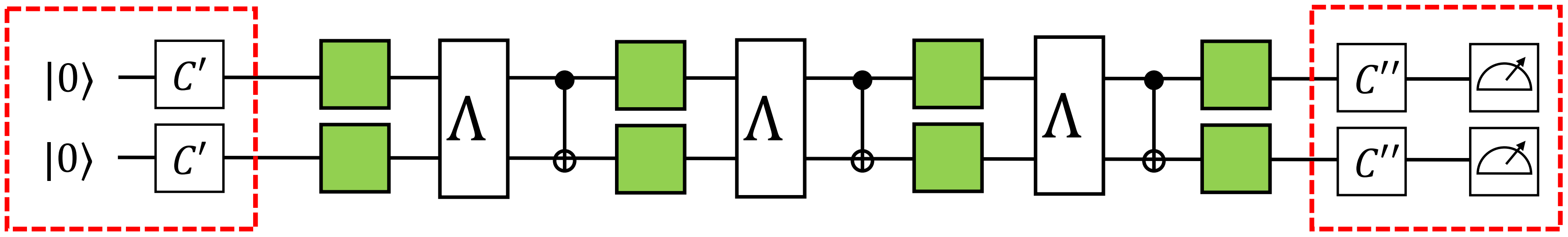


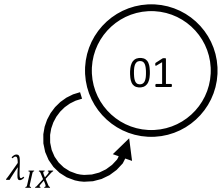
$$\mathbb{E}_{\tilde{\rho}_0}\langle\widetilde{IX}\rangle = \lambda_{IX}^3 \xi_{\mathrm{S},IX} \xi_{\mathrm{M},IX}$$

$\xi_{\mathrm{S},IX} \coloneqq \mathrm{Tr}(\tilde{\rho}_0 * IX)$ Pauli fidelity of State preparation noise

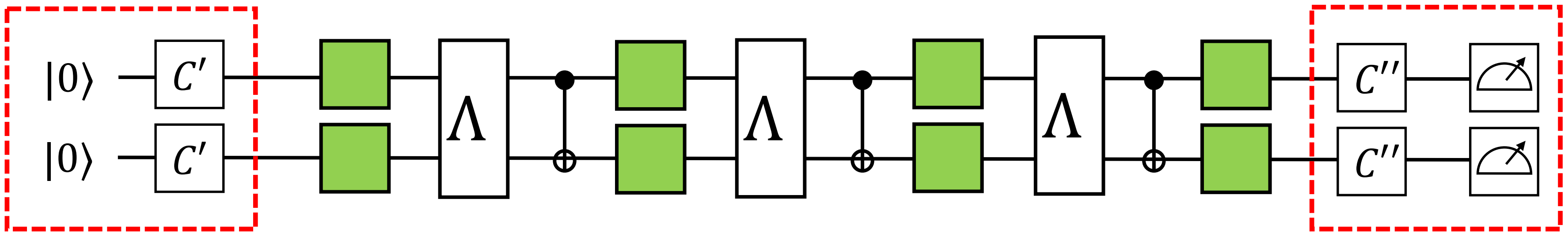
$\xi_{\mathrm{M},IX}$: Pauli Fidelity for measurement noise

Proof Sketch – Learnable part

1. For $\mathcal{G}(P_a) = P_a$, CB gives **SPAM-robust** estimate for $\lambda_a^{\mathcal{G}}$



$\mathrm{CNOT}(IX) = IX$



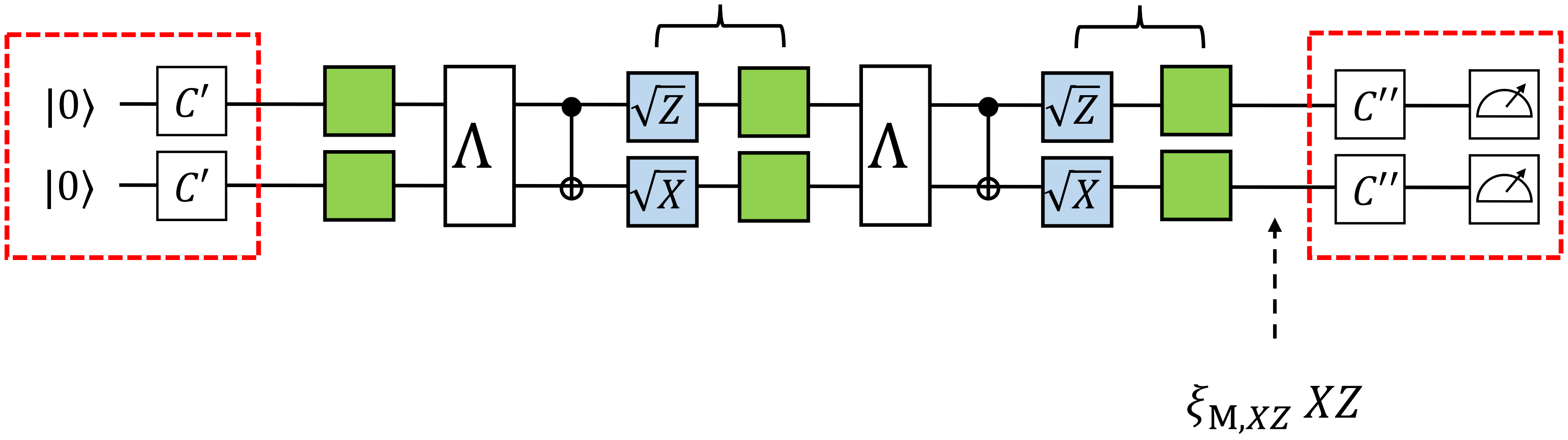
Expectation value for depth t: $F_{IX}(t) = \lambda_{IX}^t \xi_{\mathrm{S},IX} \xi_{\mathrm{M},IX}$

gate noise SPAM noise

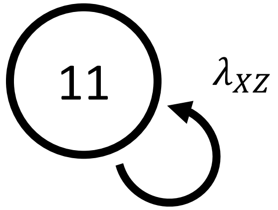
Fitting $\mathrm{F}(t)$ different t gives **SPAM-robust** estimate for λ_{IX}

Proof Sketch – Learnable part

2. If $\mathcal{G}(P_a), P_a$ have same pattern, there is 1q Clifford C such that $(C \circ \mathcal{G})(P_a) = P_a$



$\mathrm{CNOT}(XZ) = YY;$
$\sqrt{Z} \otimes \sqrt{X}(YY) = XZ$



Proof Sketch – Learnable part

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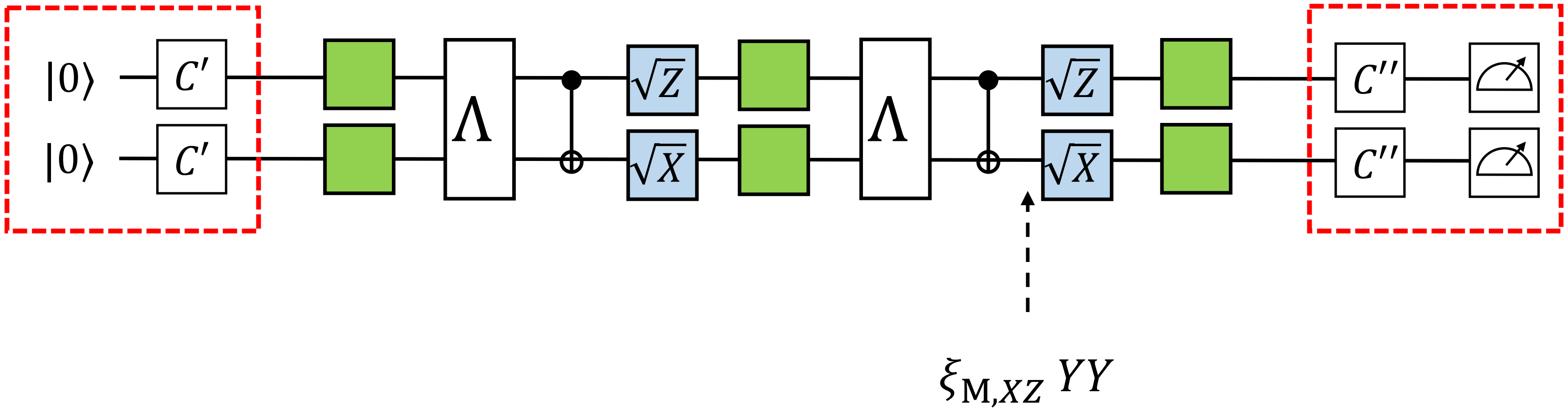

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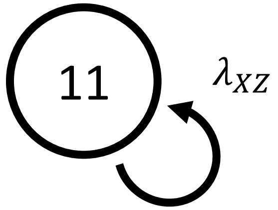

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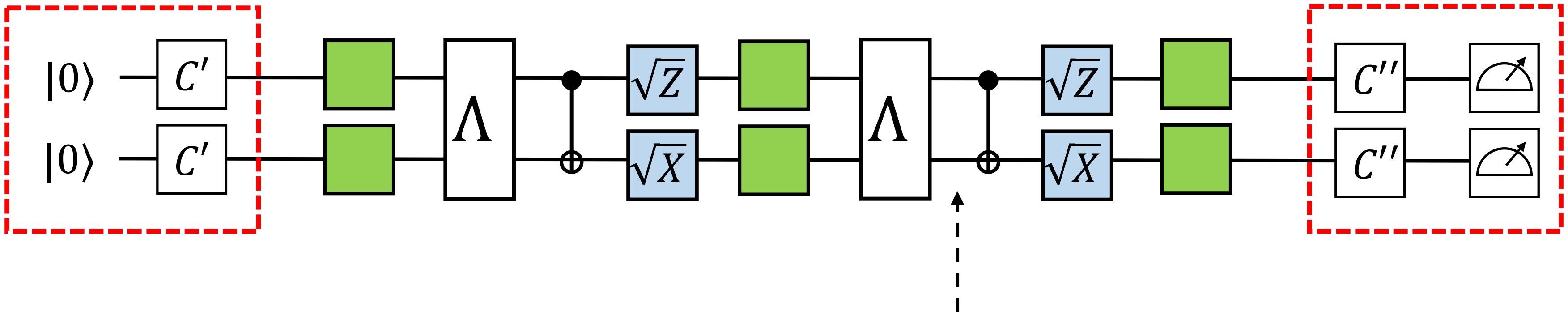

$\xi_{\mathrm{M},XZ}\ XZ$

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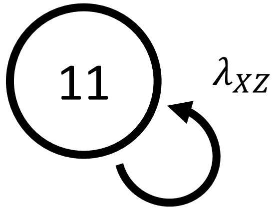

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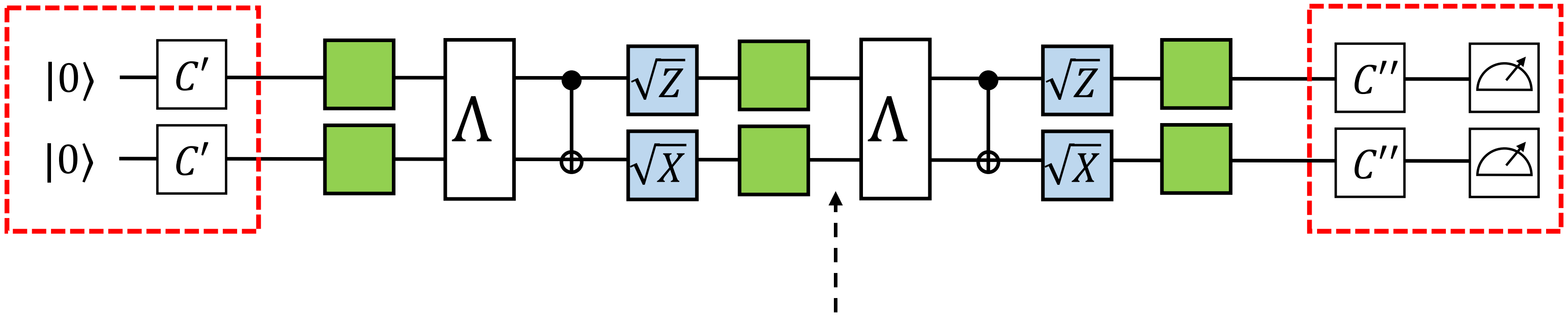
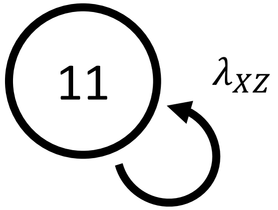


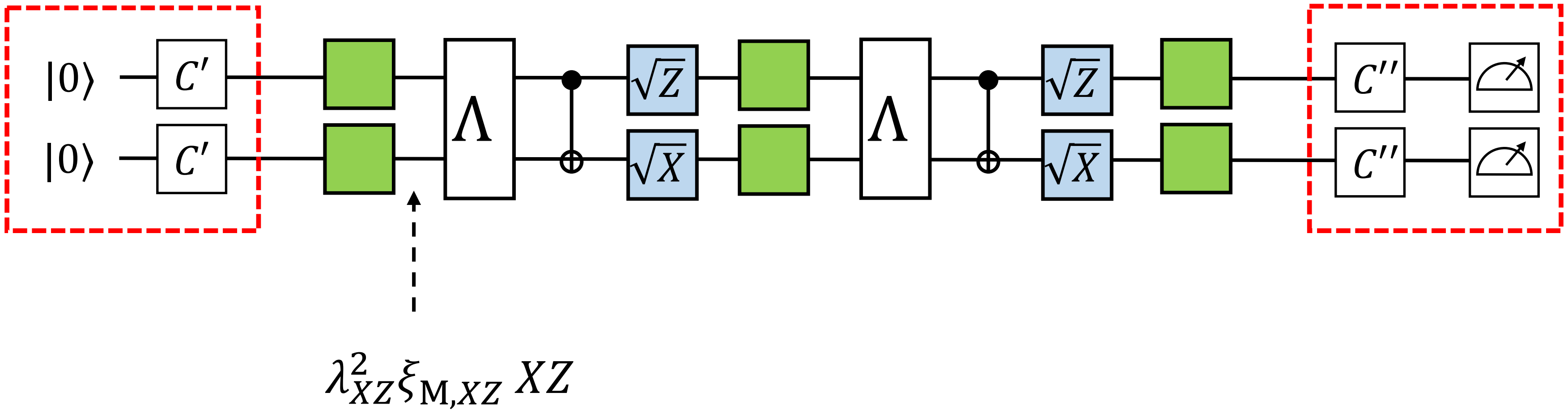
$\lambda_{XZ}\xi_{\mathrm{M},XZ}\ XZ$

$\mathrm{CNOT}(XZ) = YY;$
$\sqrt{Z} \otimes \sqrt{X}(YY) = XZ$



Proof Sketch – Learnable part

2. If $\mathcal{G}(P_a), P_a$ have same pattern, there is 1q Clifford C such that $(C \circ \mathcal{G})(P_a) = P_a$



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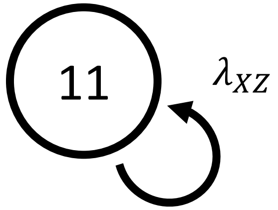

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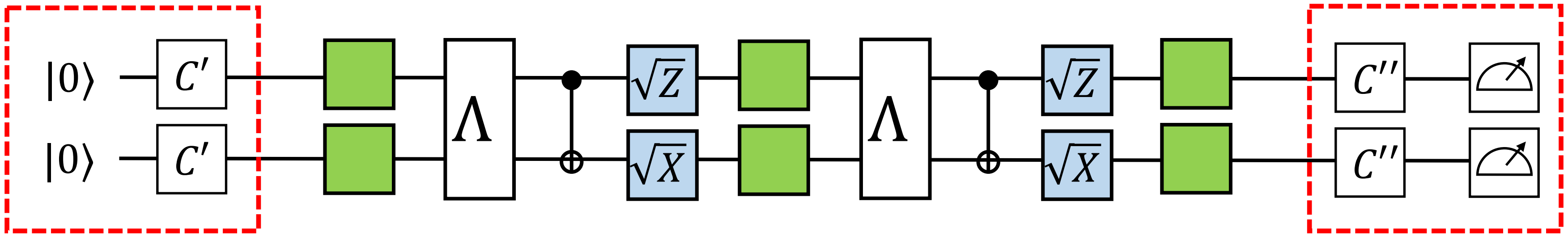

Expectation value for depth t: $F_{XZ}(t) = \lambda_{XZ}^t \xi_{\mathrm{S},XZ} \xi_{\mathrm{M},XZ}$

gate noise SPAM noise

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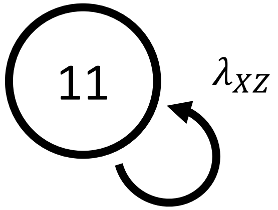
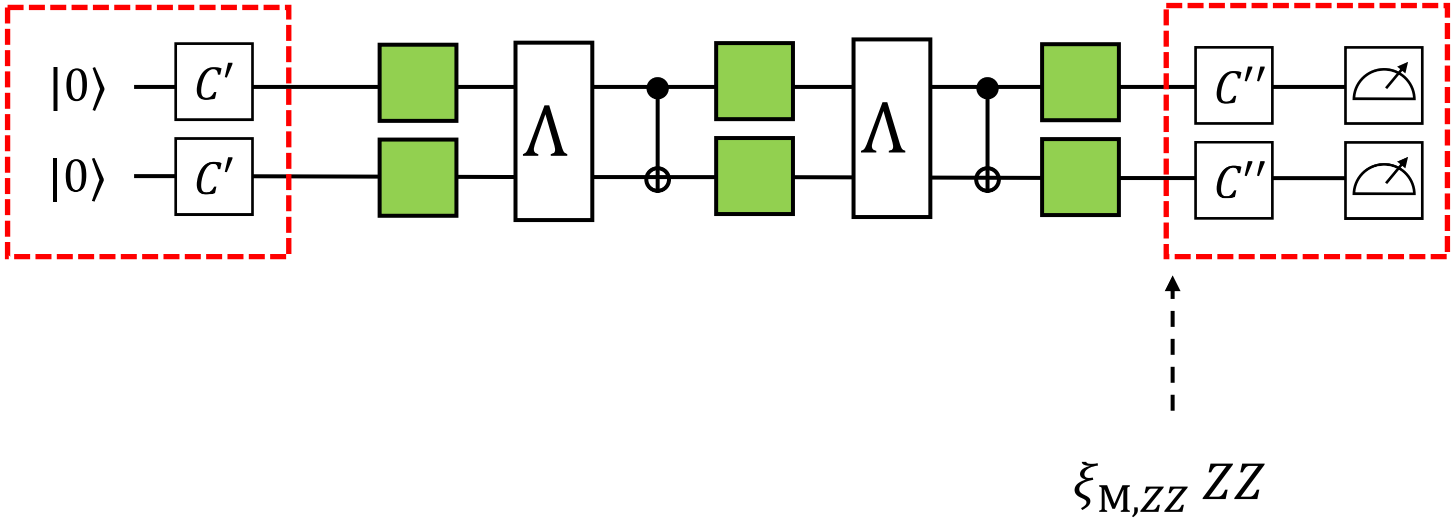
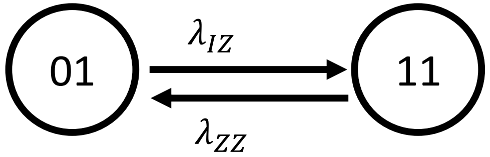


Fitting $\mathrm{F}(t)$ different t gives **SPAM-robust** estimate for λ_{XZ}

Proof Sketch – Learnable part

3. Products on any **cycle** on pattern transform graph can be estimated via CB.





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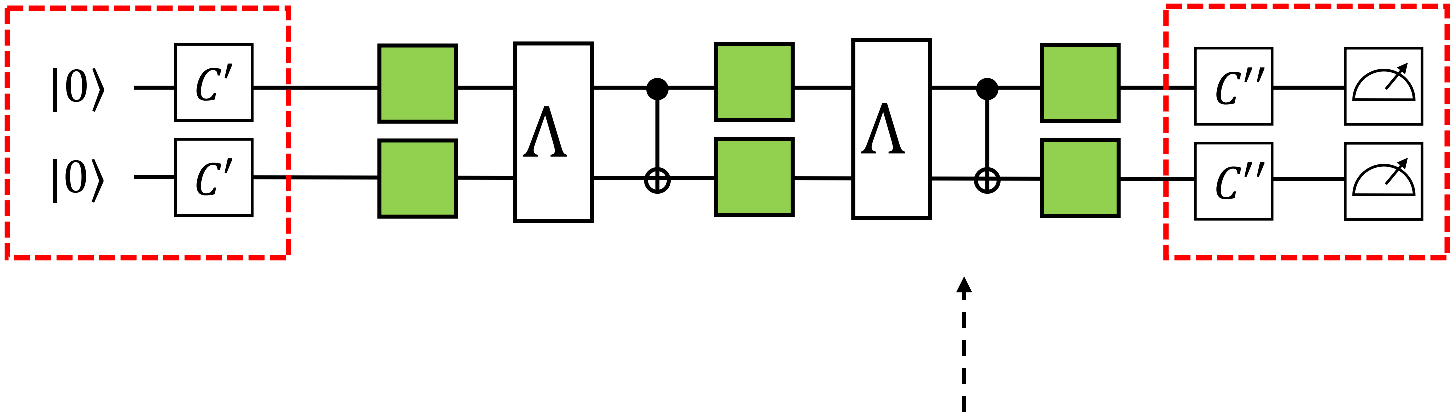
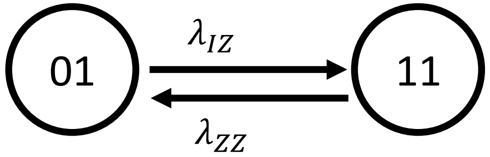


$\xi_{\mathrm{M},ZZ}\ IZ$



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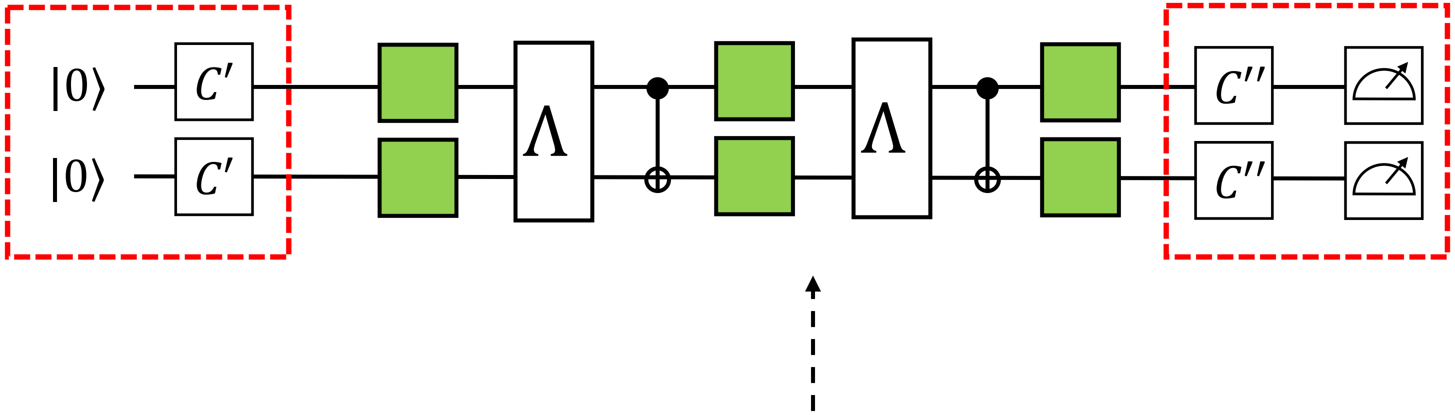
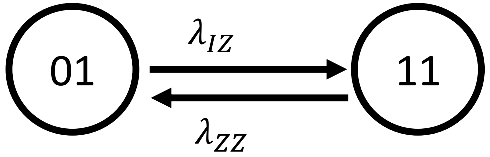


$\lambda_{IZ}\xi_{\mathrm{M},ZZ}\ IZ$



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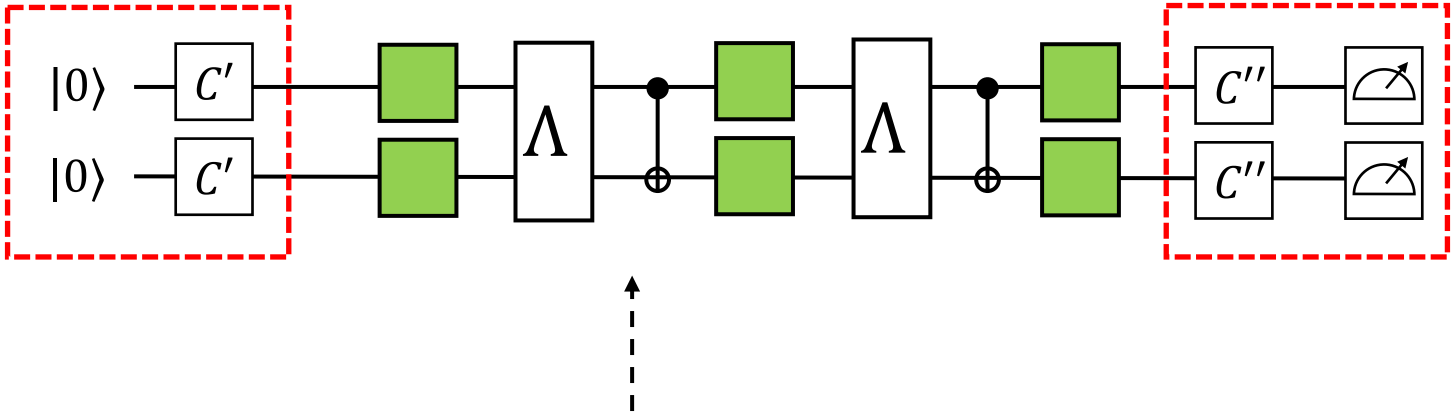

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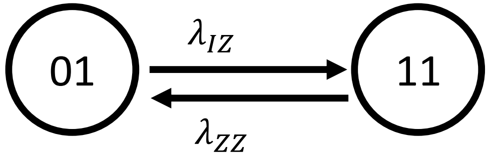

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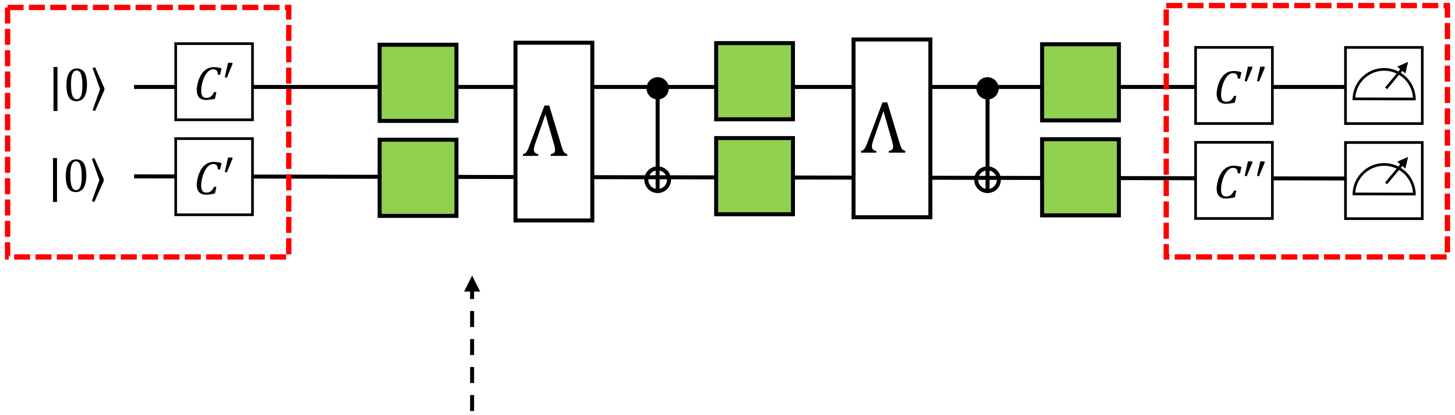
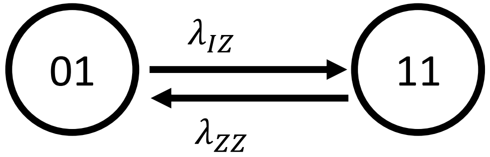


$\lambda_{ZZ}\lambda_{IZ}\xi_{\mathrm{M},ZZ}\ ZZ$



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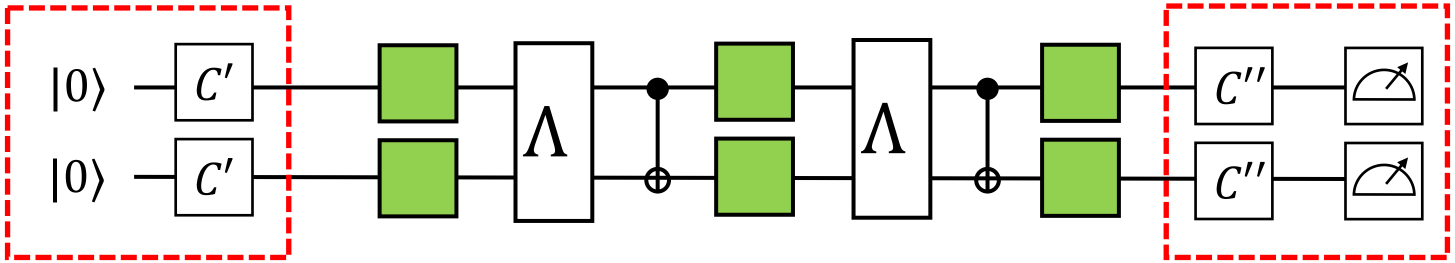
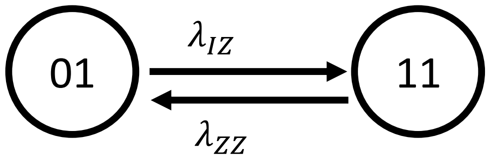


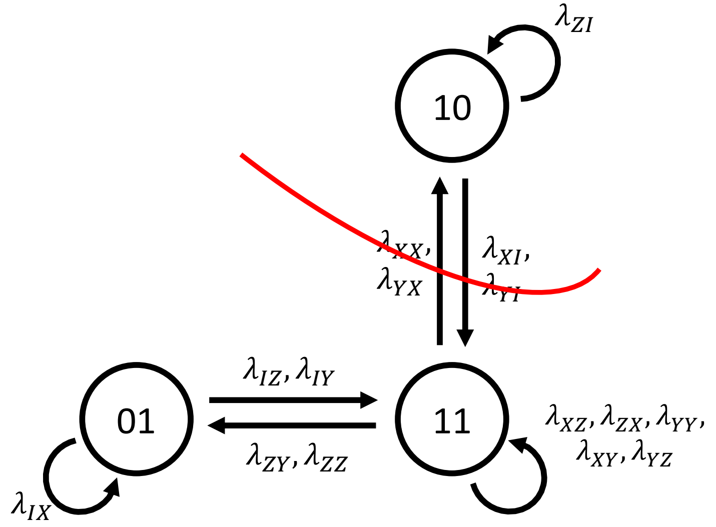


Expectation value for depth $2t$: $F_{YY}(2t) = \underbrace{\lambda_{ZZ}^t \lambda_{IZ}^t}_{\text{gate noise}} \underbrace{\xi_{\mathrm{S},ZZ} \xi_{\mathrm{M},ZZ}}_{\text{SPAM noise}}$

Fitting $\mathrm{F}(t)$ different t gives **SPAM-robus**t estimate for $\sqrt{\lambda_{ZZ}\lambda_{IZ}}$

Proof Sketch – Unlearnable part

1. We show any **cut** of pattern transform graph induces a valid **gauge transform** (s.t. Pauli noise model is preserved)



Gauge transform induced:

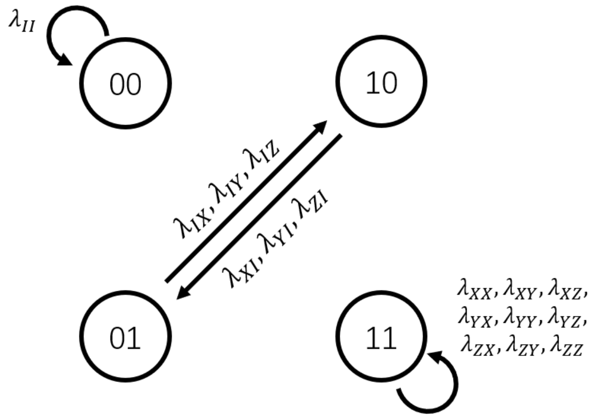
$[\lambda_{XI}, \lambda_{YI} *= \eta; \ \lambda_{XX}, \lambda_{YX} /= \eta]$

Note: SPAM noise $\xi_{S/M}$ needs to be transformed correspondingly

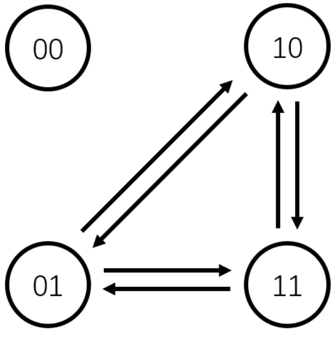
2. Using the fact that **cycle space** and **cut space** are **orthogonal complement**, we conclude only functions lies in the cycle space is learnable

   (moreover, they can be learned via “cycle” benchmarking)

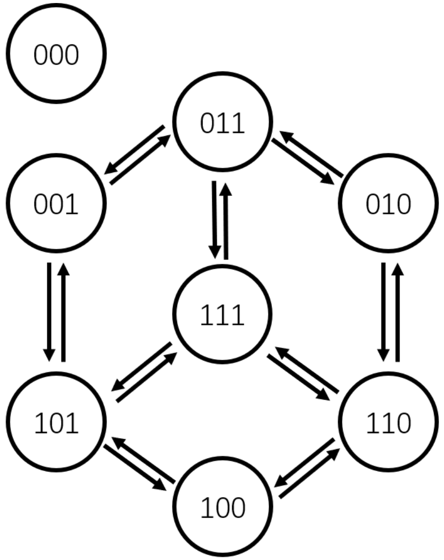
More examples



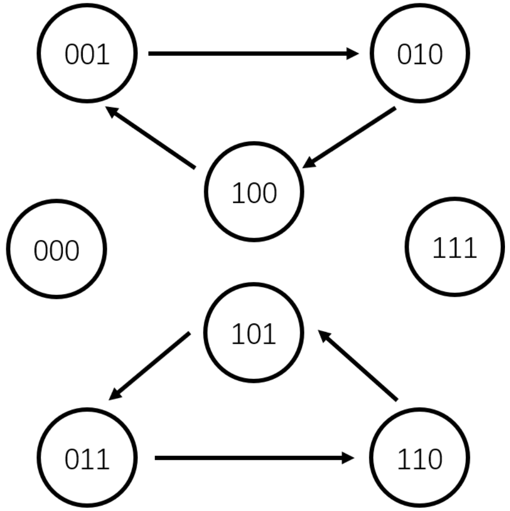
SWAP



{CNOT, SWAP}



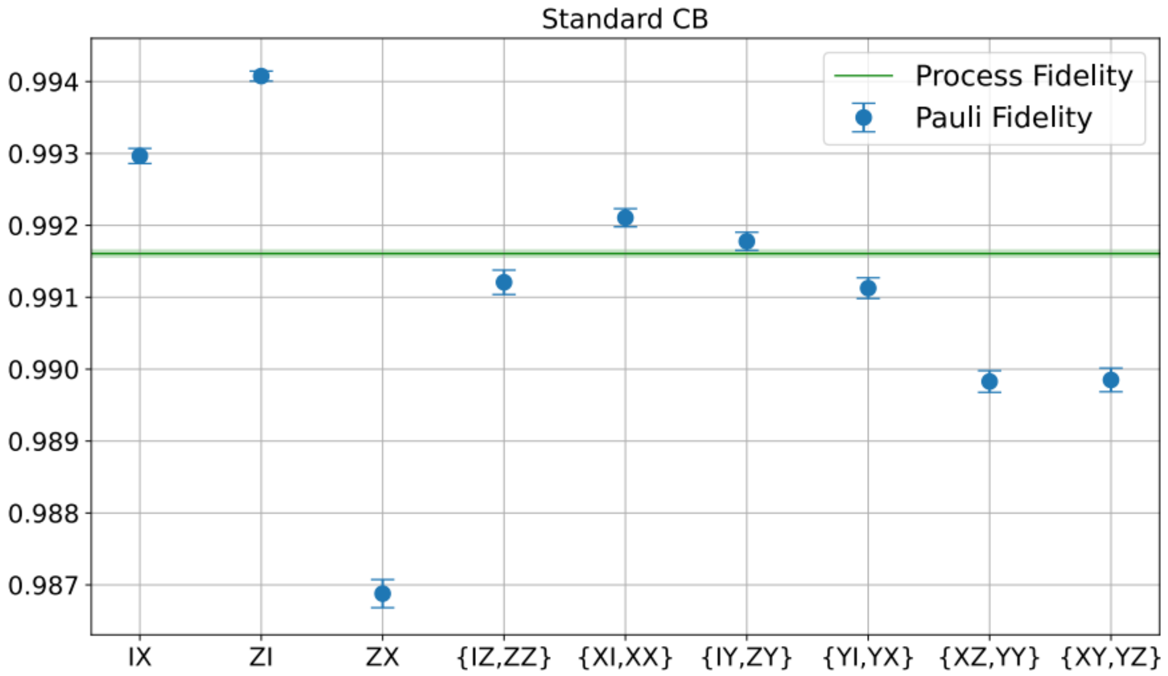
{CNOT_{12}, CNOT_{23}, CNOT_{31}}

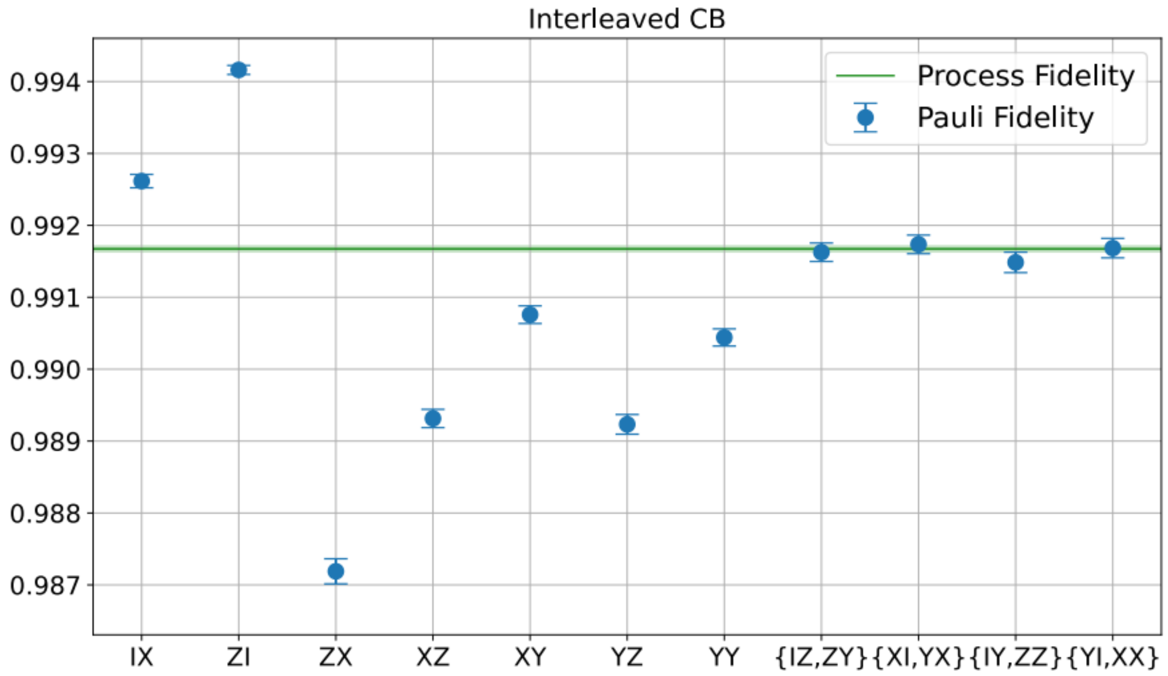


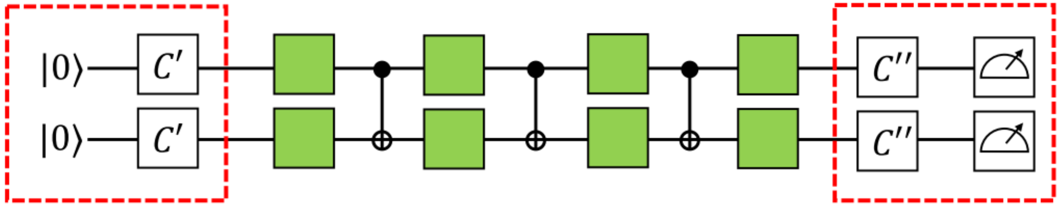
CIRC_3

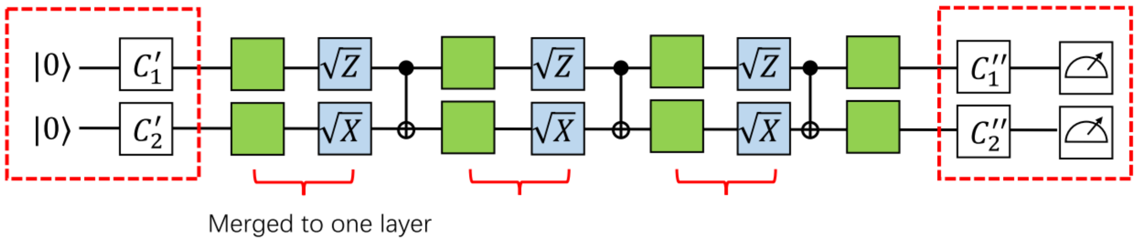
Number of qubits n	Gate set $\mathfrak{G}$	Number of parameters $\lvert\Lambda\rvert = 4^n \lvert\mathfrak{G}\rvert$	UDF($\mathfrak{G}$)
2	CNOT	16	2
2	SWAP	16	1
2	{CNOT, SWAP}	32	2
3	$\{\text{CNOT}_{12}, \text{CNOT}_{23}, \text{CNOT}_{31}\}$	192	6
3	CIRC_3	64	4

Experiments with IBM – all learnable info





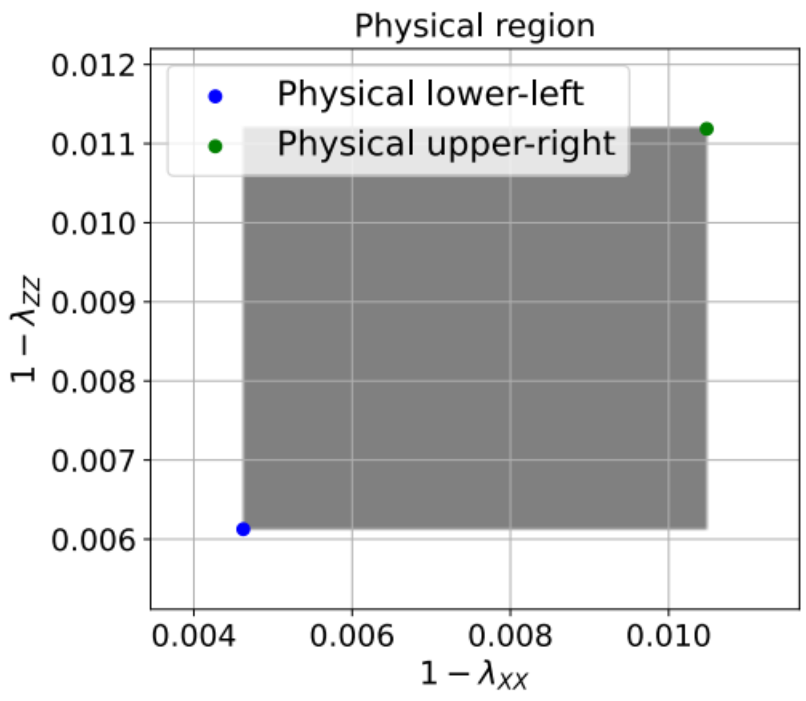


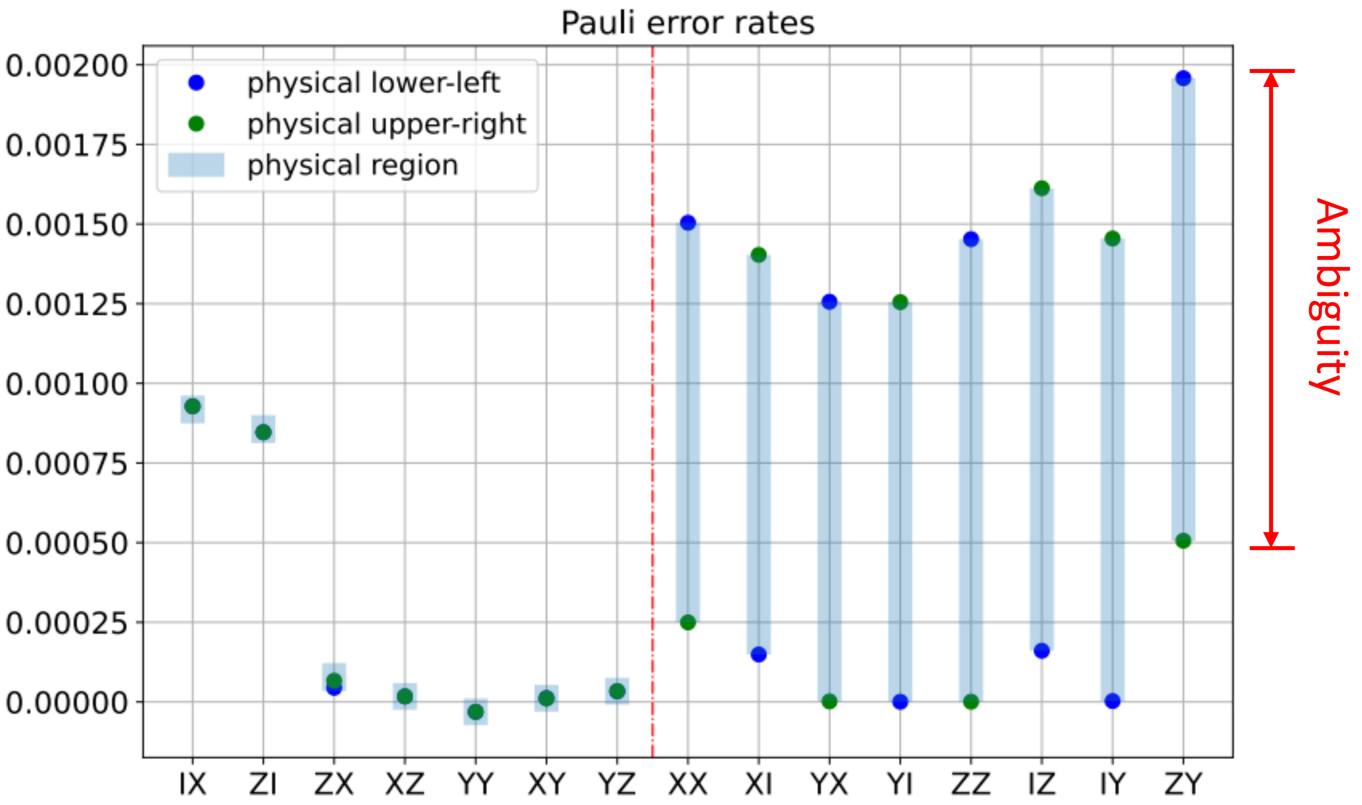


- Data taken from from `ibmq_montreal` on 2022-03-23

Experiments – Physical region

By physical constraints (quantum channel being CPTP), we can bound the unlearnable Pauli fidelities

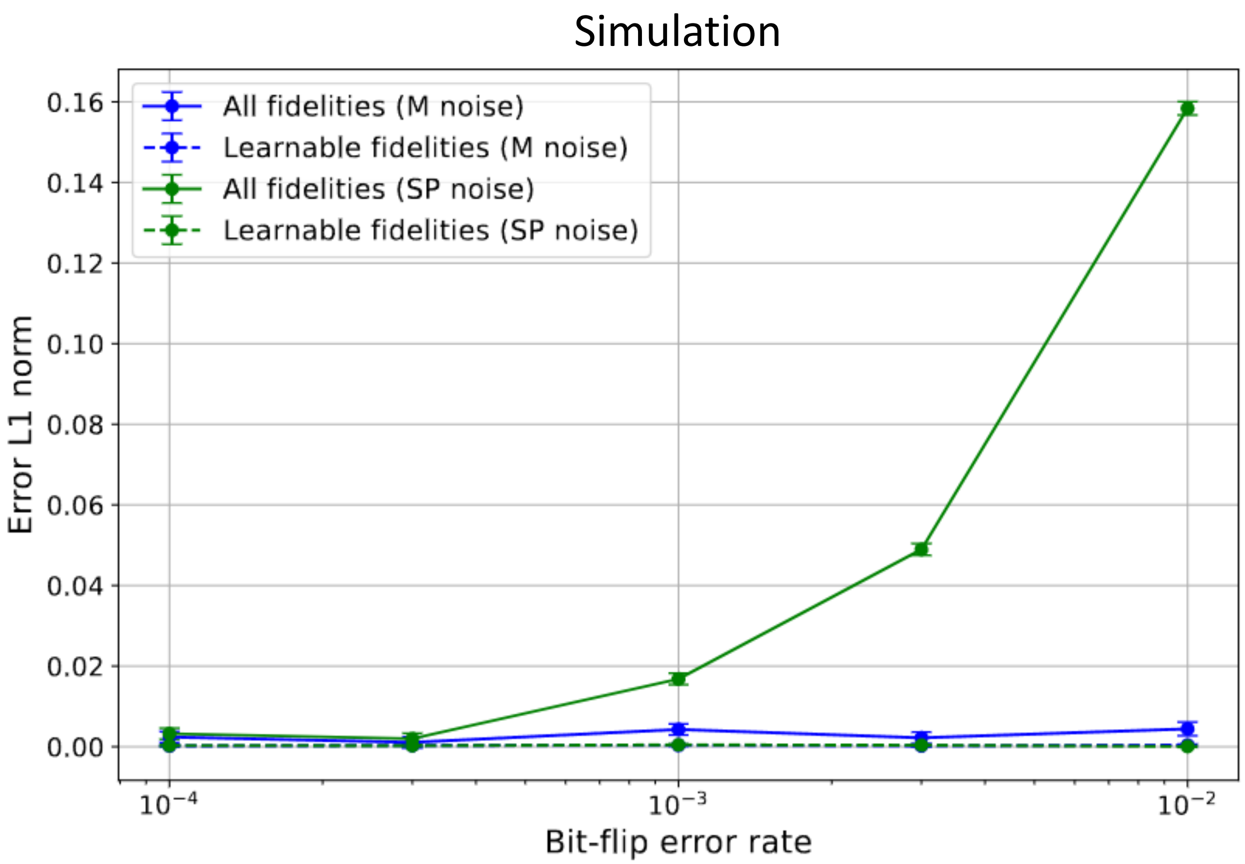




Observation: Large ambiguity for unlearnable Pauli fidelities (error rates).

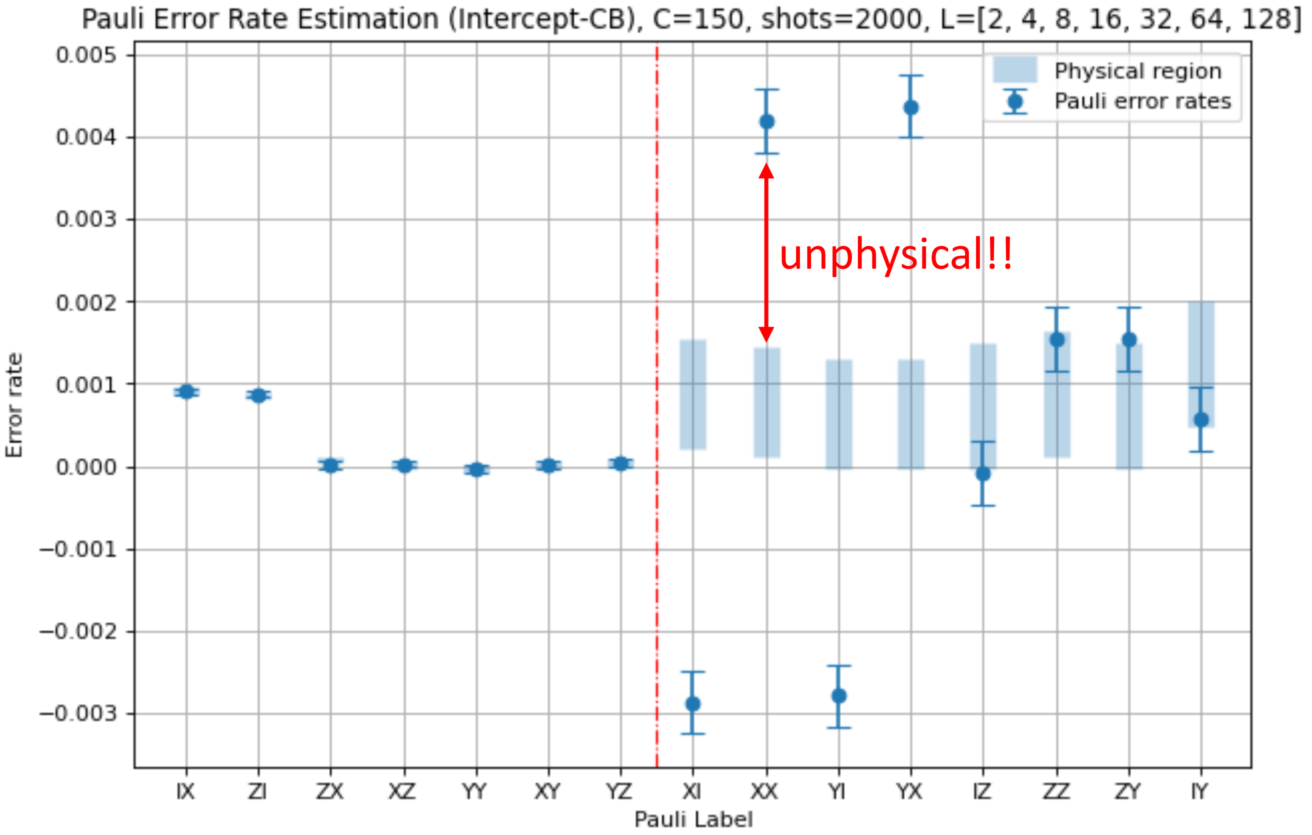
Can we resolve unlearnability?

- Unlearnability roots from gauge freedom
- Unlearnability does not exist if the initial state is **perfect**
  - No room for gauge transform
- We design protocols to learn all Pauli fidelities given **perfect SP**



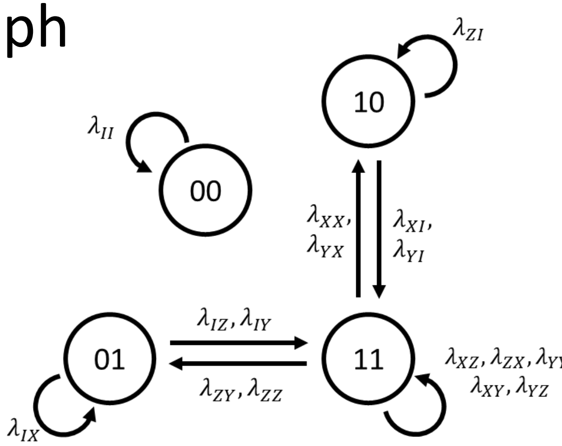
Experiments – perfect SP?

- Got unphysical results!
- This implies the assumptions of **perfect SP** is NOT practical
- Violation of physical constraints can be used to lower bound SP noise $\sim 0.6\%$
  - Could also come from other imperfectness of assumptions



Summary & Outlook

- We study the issue of learnability for Pauli noise
  - Main result: Mapping noise learnability to pattern transform graph
  - Cycle benchmarking (with trick) learns all learnable information
  - The issue of unlearnability is experimentally relevant



- Open questions
  - Learnability-consistent quantum error mitigation
  - Time/Sample complexity for learning (see e.g., [SC *et al*., Phys. Rev. A. 105, 032435])
  - Combating unlearnability by going beyond qubits or circuit model

Thank you!

- Collaborators:

Yunchao Liu, (UC Berkeley). Alireza Seif (UChicago -> IBM Quantum).

Matthew Otten (HRL Laboratories). Bill Fefferman, Liang Jiang (UChicago).

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