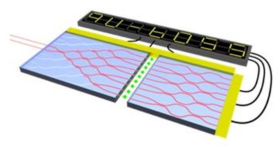
Experimental certification of contextuality, coherence and dimension in a programmable universal photonic processor



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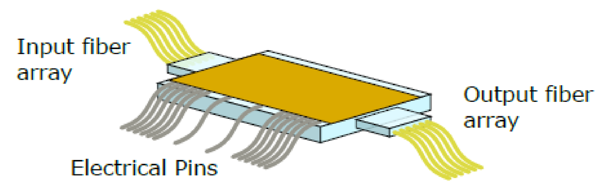
We experimentally certify coherence witnesses tailored for quantum systems of increasing dimension, using pairwise overlap measurements enabled by a universal programmable six-mode integrated laser-written chip. We show the effectiveness of the proposed coherence and dimension witnesses for qudits of dimensions up to 5. We also demonstrate advantage in a quantum interrogation task, and show it is fueled by quantum contextuality.



THEORETICAL FRAMEWORK

Bargamann invariants are witnesses of set-coherence for a set of states

- **Set-Coherence:** Any set of states $\underline{\rho} \in \{\rho_i\}$, $i\in[0,n-1]$, is said to be basis-independent coherent, or simply set-coherent, if there exists no unitary U such that $\underline{\rho} \to \underline{\sigma} = \underline{U\rho U^\dagger} = \{U\rho_i U^\dagger\}$, with σ_i diagonal for all i.

- **Graph approach** [1]: (G, r) with vertices given by quantum states and edges having weights that are two-state overlaps $r_{i,j} := \mathbf{Tr}(\rho_i\rho_j)$, for $\rho_i,\rho_j \in \rho$. It is possible to bound with linear inequalities all tuples resulting from states that are diagonal with respect to *some* basis. Such inequalities are defined recursively:

$$h_n(r) = h_{n-1}(r) + r_{0,n-1} - \sum_{i=1}^{n-2} r_{i,n-1} \leq 1 \qquad (1).$$

  starting with $h_3(r) = r_{0,1} + r_{0,2} - r_{1,2}$ that is also witness of contextuality in the interpretation where each is an operational preparation procedure as well as a measurement effect.

- Using multi-mode devices it is possible to witness coherence achievable with **qudits only** by violations of (1), as well as Hilbert space dimension higher than two.

Robust efficiency for the task of Quantum Interrogation [2]

- Quantum interrogation: Using as many photons as needed, detect the presence of an photo-sensitive bomb without exploding it, with the highest possible probability.

- Balanced Mach-Zehnder interferometer with a possible bomb placed in one arm: there is a 25% probability to detect the presence of the object without directly interacting with it. Allowing the MZI beam-splitting ratios to differ from 1/2, one can quantify the success rate using the efficiency η given by:

$$\eta = \frac{p_{success}}{p_{success}+p_{absorption}} = \frac{r(1-r)}{r(1-r)-r+1} \qquad (2).$$

  As shown in [3], noncontextual models cannot explain η for arbitrary beam-splitting ratios, and there exists a quantifiable gap between the efficiencies achievable by quantum theory and noncontextual models.

COHERENCE AND CONTEXTUALITY IN 2-LEVEL SYSTEMS

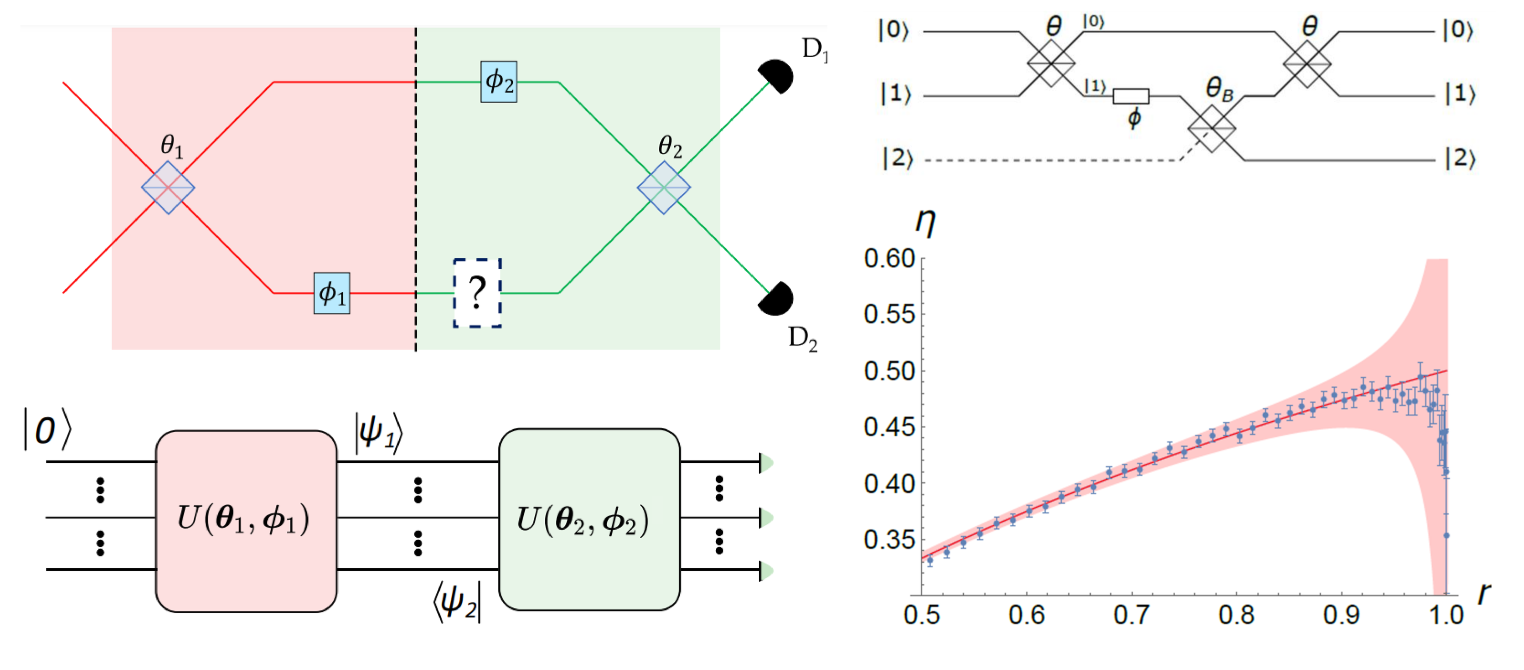


Figure 1-left: Mach-Zehnder interferometer, ideally separated in a preparation stage (red), in which we prepare a qubit state $|\psi(\theta_1,\phi_1)\rangle$, and a measurement stage (green) that projects onto another qubit state $|\psi(\theta_2,\phi_2)\rangle$. In a quantum interrogation experiment, the ?-box is an object that absorbs photons. In analogy with the MZI, a multi-path interferometer encodes d-level systems, With a single-photon input at the top input mode $|0\rangle$, this setup is capable of measuring two-state overlaps $|\langle\psi_2|\psi_1\rangle|^2 = |\langle 0|U(\theta_2,\phi_2)^\dagger U(\theta_1,\phi_1)|0\rangle|^2$
Figure 1-right: Portion of the circuit employed for the quantum interrogation task. Efficiency η of the object's detection versus the reflectivity $r=\sin\theta$ of the two BSs in the MZI. The red curve is the theoretical prediction while the red shaded area represents the deviations from the ideal model due to dark counts and imperfect calibration of the BSs. The error bars derive from the poissonian statistics of the single-photon counts

COHERENCE AND DIMENSION WITNESSES IN $d>2$

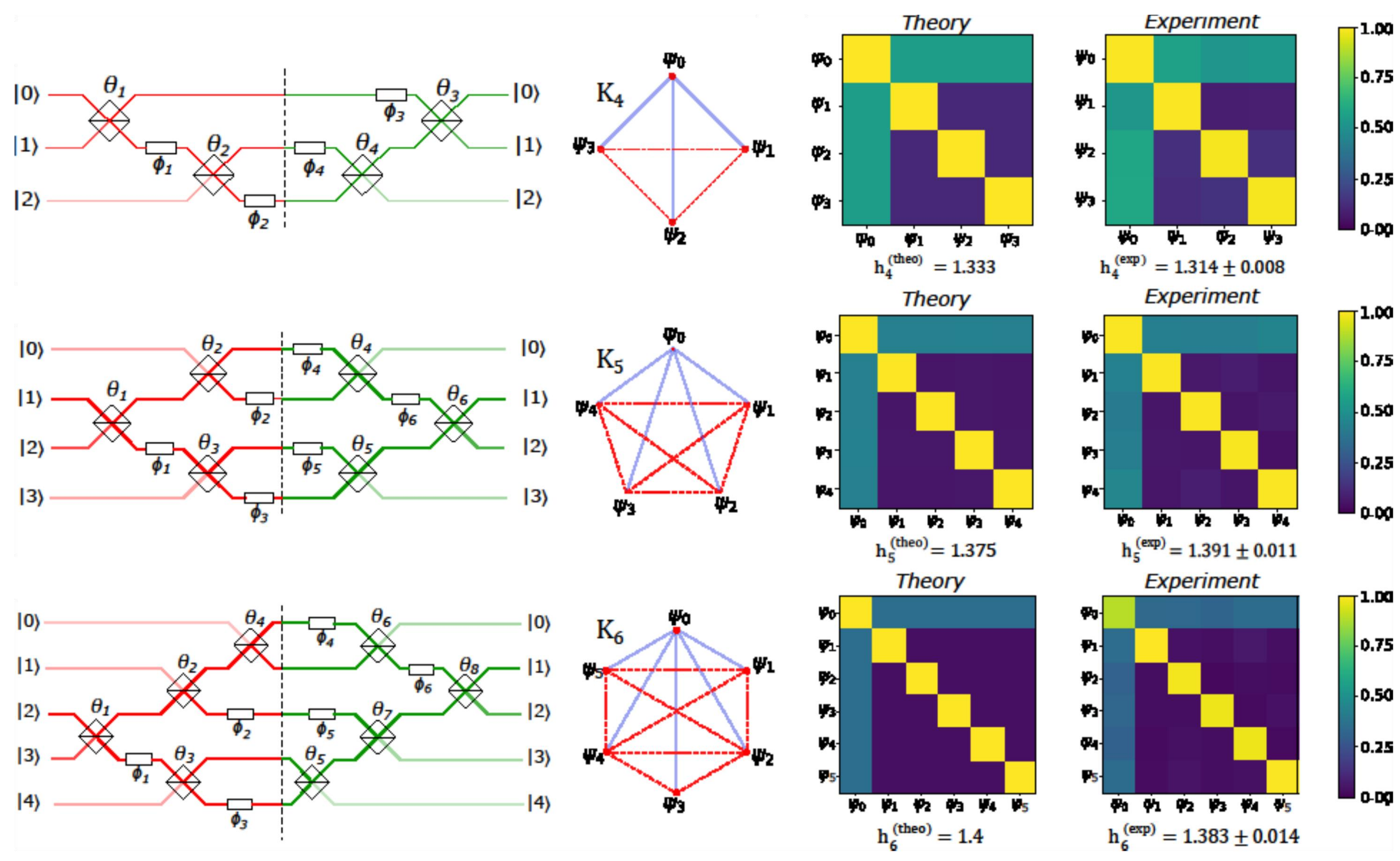


Figure 2: Circuit scheme of the qutrits preparation (red) and measurement (green), with the graphs K_i $(i=4,5,6)$ identifying the inequalities h_i. Such inequalities are violated by sets of i qutrit states $\{\psi_0, \ldots \psi_i\}$. On the right, theoretical and experimental matrices of the mutual overlaps between the states of the set that has the maximum violation of the h_i inequality.

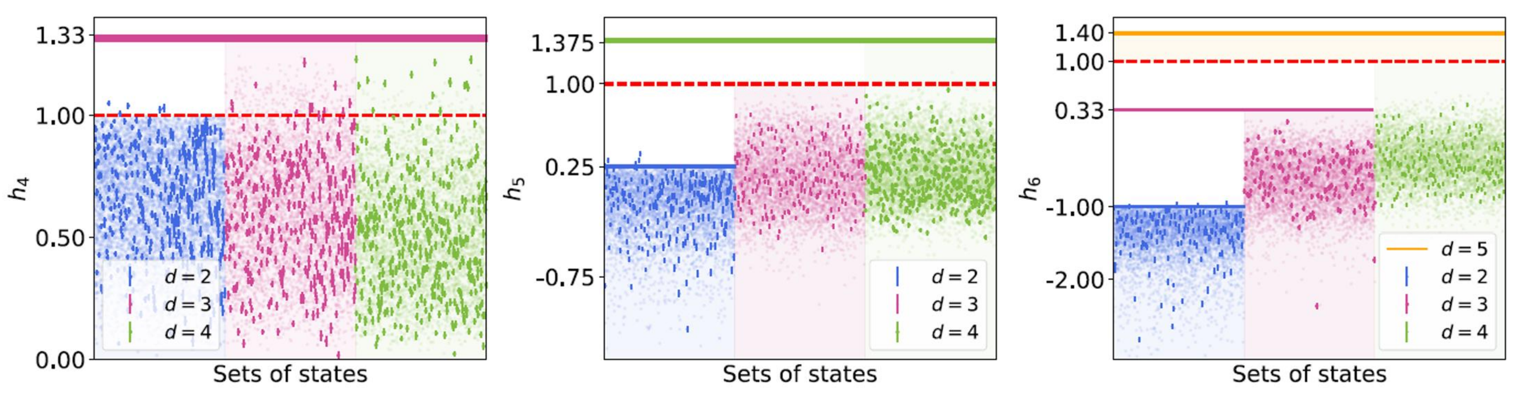


Figure 3: h_n inequalities as dimension witnesses. Random states sampled uniformly in Hilbert spaces of dimension 2 (blue), 3 (purple) and 4 (green). Bold points correspond to ~ 200 extractions of sets of random qudits measured in the experiment for each dimension. The shaded points in the background are ~ 5000 sets of numerically simulated states for the different dimensions. In the case of h_6 we report only the maximum violation measured for the coherence witness since our setup is not a universal generator of states with $d = 5$.

CONCLUSIONS

- Presence of basis-independent coherence generated inside a MZI, by achieving the largest violation up to date of coherence witness inequalities.
- Presence of generalized contextuality within the MZI by violation of a recently introduced novel generalized noncontextuality inequality.

- $h_n(r)$ provide information regarding the coherence accessible only due to the dimension of the space, since they are not violated by sets of states without coherence, or by systems with dimension $d < n - 1$.
- In Fig. 3, it is clear that the maximum violations are not typically achievable when sampling random states. However, the h_n become very effective in discerning systems of $d > 2$ for increasing values of n.

We believe that violation of these inequalities can be exploited in the future as a novel kind of witness of coherence, and may be related to the hardness of quantum computation.
Moreover, the theoretical results presented here apply to any platform for quantum computation, and not just photonics.

References:
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