Reconquering Bell sampling on qudits: stabilizer learning and testing, quantum pseudorandomness bounds, and more

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Stabiliser states are central to quantum computation, and find applications in quantum error correction [Sho95, CS96, Got96, Got97], efficient classical simulations of quantum circuits [AG04, BSS16, BBC+19], low-rank recovery [KZG16], randomized benchmarking [KLR+08, MGE11, HWFW19], measurement-based quantum computation [RB00], tensor networks for holography codes [HNQ+16, NW20], and quantum learning algorithms [HKP20]. A major tool in exploring stabiliser states is *Bell sampling* [Mon17], which involves measuring two copies of a given n-qubit state $|\psi\rangle \in (\mathbb{C}^2)^{\otimes n}$ in the Bell basis. In spite of its simplicity, Bell sampling – along with a variant called Bell difference sampling [GNW21] – has proved to be a powerful primitive, and has found applications in learning unknown stabilizer states, testing the stabilizer fidelity and dimension of states, and discriminating between the outputs of Haar-random circuits and Clifford circuits doped with non-Clifford gates [Mon17, GNW21, HG24, GIKL25, GIKL24b, GIKL24a].

While Bell (difference) sampling and its applications were originally proposed for qubits, it was not known how to generalise the method to d-level systems, i.e., *qudits*, in a meaningful and useful way, an issue which we resolve in this work.

To explain Bell (difference) sampling and our generalization in more detail, let us introduce some notation and recall various concepts relating to the Pauli group on qudits. Let $\omega \triangleq \mathrm{e}^{\mathrm{i}2\pi/d}$ be the principal d-th root of unity and $\tau \triangleq (-1)^d \mathrm{e}^{\mathrm{i}\pi/d}$, which satisfies $\tau^2 = \omega$. Let $D = d$ if d is odd and $D = 2d$ if d is even. Define the unitary shift and clock operators X and Z, respectively, as $\mathsf{X}|q\rangle = |q+1\rangle$ and $\mathsf{Z}|q\rangle = \omega^q|q\rangle$ for all $q \in \mathbb{Z}_d$. The *Weyl operators on n qudits are defined as*

$$\mathcal{W}_{\mathbf{x}} = \mathcal{W}_{\mathbf{v},\mathbf{w}} \triangleq \tau^{\langle \mathbf{v},\mathbf{w}\rangle}(\mathsf{X}^{v_1}\mathsf{Z}^{w_1}) \otimes \cdots \otimes (\mathsf{X}^{v_n}\mathsf{Z}^{w_n}) \qquad \forall \mathbf{x} = (\mathbf{v},\mathbf{w}) \in \mathbb{Z}_d^{2n},$$

where $\langle \mathbf{v},\mathbf{w}\rangle = \sum_{i=1}^n v_i w_i$.

The n-qudit Pauli group $\mathscr{P}_d^n \triangleq \{\tau^s \mathcal{W}_{\mathbf{x}} : \mathbf{x} \in \mathbb{Z}_d^{2n}, s \in \mathbb{Z}\}$ is the group of all Weyl operators up to a power of τ, while the n-qudit Clifford group $\mathscr{C}_d^n \triangleq \{\mathcal{U} \in \mathbb{C}^{d^n \times d^n} : \mathcal{U}^\dagger \mathcal{U} = \mathbf{I}, \mathcal{U}\mathscr{P}_d^n\mathcal{U}^\dagger = \mathscr{P}_d^n\}$ is the group of $d^n \times d^n$ unitary operators that normalise the Pauli group. A stabilizer group $\mathcal{S} \subset \mathscr{P}_d^n$ is a maximal subgroup of the n-qudit Pauli group which contains only the identity from the center of $\mathscr{P}_d^n$. The generalised Bell basis states are then defined as

$$|\mathcal{W}_{\mathbf{x}}\rangle \triangleq (\mathcal{W}_{\mathbf{x}} \otimes \mathbb{I})|\Phi^+\rangle, \qquad \text{where} \quad |\Phi^+\rangle \triangleq d^{-\frac{n}{2}} \sum_{\mathbf{q}\in\mathbb{Z}_d^n} |\mathbf{q},\mathbf{q}\rangle \quad \text{is a maximally entangled state.}$$

Bell sampling on qubits [Mon17] measures two copies of an n-qubit state $|\psi\rangle = \sum_{\mathbf{q}\in\mathbb{F}_2^{2n}} \alpha_{\mathbf{q}}|\mathbf{q}\rangle$ in the Bell basis $|\mathcal{W}_{\mathbf{x}}\rangle$ to obtain a string $\mathbf{x} \in \mathbb{F}_2^{2n}$ from the distribution $2^{-n}|\langle\psi|\mathcal{W}_{\mathbf{x}}|\psi^*\rangle|^2$, where $|\psi^*\rangle = \sum_{\mathbf{q}\in\mathbb{F}_2^{2n}} \alpha_{\mathbf{q}}^*|\mathbf{q}\rangle$ is the complex conjugate of $|\psi\rangle$. Bell difference sampling [Mon17, GNW21] performs Bell sampling twice and computes the difference of the two outcome strings. When applied to $|\psi\rangle^{\otimes 4}$, Bell difference sampling returns a string $\mathbf{x} \in \mathbb{F}_2^{2n}$ with probability

$$q_\psi(\mathbf{x}) \triangleq \sum_{\mathbf{y}\in\mathbb{F}_2^{2n}} p_\psi(\mathbf{y}) p_\psi(\mathbf{x}-\mathbf{y}), \qquad \text{where} \quad p_\psi(\mathbf{x}) \triangleq 2^{-n}|\langle\psi|\mathcal{W}_{\mathbf{x}}|\psi\rangle|^2$$

is called the *characteristic distribution*. When $|\psi\rangle$ is a stabilizer state $|\mathcal{S}\rangle$ of some stabilizer group $\mathcal{S}$, then $q_{\mathcal{S}}(\mathbf{x}) = 2^{-n}$ if $\mathcal{W}_{\mathbf{x}} \in \mathcal{S}$ (up to some phase) and 0 otherwise. In other words, Bell

difference sampling on copies of a stabiliser state returns a uniformly random element of $\mathcal{S}$, and that is where its power lies.

As mentioned, the generalization of Bell (difference) sampling to d-level systems has proved challenging. Indeed, the authors recently showed that the natural extension of the qubit scheme, based on measuring two copies of a given n-qudit state $|\psi\rangle \in (\mathbb{C}^d)^{\otimes n}$ in the generalised Bell basis, can return a uniformly random string, and thus may provide no information about the corresponding stabilizer group at all [ADIS24].

In this work, we extend Bell sampling to qudits for all dimensions d such that the useful properties on qubits are carried over. Our generalisation is non-trivial yet simple, and at its heart it is a novel unitary that maps four copies $|\mathcal{S}\rangle^{\otimes 4}$ of a stabiliser state to their complex conjugate $|\mathcal{S}^*\rangle^{\otimes 4}$ (up to some qudit Pauli operator), which may be of independent interest. The unitary we introduce is based on Lagrange's four-square theorem, which states that every positive integer can be written as the sum of four integer squares.

Result 1. *For every integer $d \geqslant 2$, let a_1, a_2, a_3, a_4 be non-negative integers such that $a_1^2 + a_2^2 + a_3^2 + a_4^2 = D - 1$. Define the unitary $\mathcal{B}_{\mathbf{R}} \in ((\mathbb{C}^d)^{\otimes n})^{\otimes 4}$*

$$\mathcal{B}_{\mathbf{R}} : |\mathbf{Q}\rangle \mapsto |\mathbf{QR} \pmod d\rangle \text{ where } \mathbf{Q} = [\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4] \in \mathbb{Z}_d^{n\times 4} \text{ and } \mathbf{R} = \begin{pmatrix} a_1 & a_2 & a_3 & a_4 \\ a_2 & -a_1 & a_4 & -a_3 \\ a_3 & -a_4 & -a_1 & a_2 \\ a_4 & a_3 & -a_2 & -a_1 \end{pmatrix}.$$

Given any stabiliser state $|\mathcal{S}\rangle \in (\mathbb{C}^d)^{\otimes n}$, then $\mathcal{B}_{\mathbf{R}}|\mathcal{S}\rangle^{\otimes 4} = \mathcal{P}(\mathcal{S})|\mathcal{S}^*\rangle^{\otimes 4}$ for some Pauli operator $\mathcal{P}(\mathcal{S})$ that depends on $|\mathcal{S}\rangle$.

The crucial property behind the matrix $\mathbf{R}$ that guarantees the transformation of $|\mathcal{S}\rangle^{\otimes 4}$ into $|\mathcal{S}^*\rangle^{\otimes 4}$ is that $\mathbf{R}^\top\mathbf{R} = \mathbf{R}\mathbf{R}^\top = (D-1)\mathbf{I}$. It is well known that stabiliser states are quadratic phase states [HDDM05], meaning they are of the form $|\mathscr{A}|^{-\frac{1}{2}} \sum_{\mathbf{q}\in\mathscr{A}} \omega^{L(\mathbf{q})}\tau^{Q(\mathbf{q})}|\mathbf{q}\rangle$ where $\mathscr{A}$ is some affine submodule of $\mathbb{Z}_d^n$ and $L, Q : \mathbb{Z}^n \to \mathbb{Z}$ are linear and quadratic polynomials. The unitary $\mathcal{B}_{\mathbf{R}} : |\mathbf{Q}\rangle \mapsto |\mathbf{QR} \pmod d\rangle$ introduces, by a change of variables, the matrix $\mathbf{R}$ into the joint phases of $|\mathcal{S}\rangle^{\otimes 4}$, and the factor $\mathbf{R}^\top\mathbf{R} = (D-1)\mathbf{I}$ appears in the quadratic polynomial $Q(\mathbf{Q})$, mapping it to $(D-1)Q(\mathbf{Q})$, and so $\tau^{Q(\mathbf{Q})} \mapsto \tau^{-Q(\mathbf{Q})}$. The complex conjugate of the linear part can be obtained by some Pauli operator $\mathcal{P}(\mathcal{S})$. Together, $\mathcal{P}(\mathcal{S})\mathcal{B}_{\mathbf{R}}|\mathcal{S}\rangle^{\otimes 4} = |\mathcal{S}^*\rangle^{\otimes 4}$.

Our generalised Bell sampling procedure then proceeds as follows: take 8 copies of a stabiliser state $|\mathcal{S}\rangle$, apply $\mathcal{B}_{\mathbf{R}}$ to 4 of the 8 copies to obtain $|\mathcal{S}\rangle^{\otimes 4}|\mathcal{S}^*\rangle^{\otimes 4}$ (up to some Pauli operators) and carry on with the standard procedure of measuring each pair $|\mathcal{S}\rangle|\mathcal{S}^*\rangle$ in the Bell basis. The result is four strings in $\mathbb{Z}_d^{2n}$. If we repeat this procedure again and take the difference between the outcomes, the four resulting strings $\mathbf{x}_1, \ldots, \mathbf{x}_4 \in \mathbb{Z}_d^{2n}$ are samples from the characteristic distribution $p_{\mathcal{S}}(\mathbf{x}) = d^{-n}|\langle\mathcal{S}|\mathcal{W}_{\mathbf{x}}|\mathcal{S}\rangle|^2$ which, as mentioned above, is uniformly distributed over the strings corresponding to the stabiliser group $\mathcal{S}$. We thus successfully sample four generators of the stabiliser group through this procedure. More generally, we prove that our generalised Bell difference sampling has a useful interpretation if we are given copies of an arbitrary (not necessarily stabiliser) state $|\psi\rangle$.

Result 2. *Given $|\psi\rangle \in (\mathbb{C}^d)^{\otimes n}$, our generalised Bell difference sampling on $|\psi\rangle^{\otimes 16}$ corresponds to sampling from the distribution*

$$b_\psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4) = \sum_{\mathbf{Y}\in\mathbb{Z}_d^{2n\times 4}} \prod_{i=1}^{4} p_\psi(\mathbf{x}_i + \mathbf{Y}_i) p_\psi((\mathbf{YR})_i), \qquad \text{where} \quad p_\psi(\mathbf{x}) \triangleq d^{-n}|\langle\psi|\mathcal{W}_{\mathbf{x}}|\psi\rangle|^2,$$

and where $\mathbf{Y}_i$ and $(\mathbf{YR})_i$ denote the i-th columns of the matrices $\mathbf{Y}$ and $\mathbf{YR}$, respectively.

We then apply our new Bell sampling primitive to lift several known results from qubits to qudits (see Table 1 for a comparison of our results with prior works):

Problem	Dimension d	Work	Sample Complexity	Time complexity	Remarks
Stabiliser learning	$d = 2$	[AG08]	$O(n^2)$	$O(n^3)$	Operations over 1 copy
		[ZPDF16]	$O(n)$	$O(n^3)$	For graph states Operations over 2 copies
		[Mon17]	$O(n)$	$O(n^3)$	Operations over 2 copies
	$d \geqslant 2$ prime	[ADIS24]	$O(n)$	$O(n^3)$	Operations over 3 copies
	$d \geqslant 2$	[HEC25]	$O(nd \log d)$	$O(n^3 \log d + nd \log d)$	Operations over D copies
		This work	$O(n)$	$O(n^3)$	Operations over 8 copies
Hidden Stabiliser Group	$d \geqslant 2$	[HEC25]	$O(n \max\{d, \varepsilon^{-1}\} \log d)$	$O((n + \frac{n^3}{d}) \max\{d, \frac{1}{\varepsilon}\} \log d)$	Operations over D copies
		This work	$O((n/\varepsilon) \min\{\sum_{i=1}^{\ell} k_i, \log p_\ell\})$	$O((n^3/\varepsilon) \min\{\sum_{i=1}^{\ell} k_i, \log p_\ell\})$	Operations over 8 copies
Stabiliser size testing	$d = 2$	[GIKL25]	$O(n/\varepsilon)$	$O(n^3/\varepsilon)$	$\varepsilon < 3/8$
	$d \geqslant 2$	This work	$O\big((n/\varepsilon) \sum_{i=1}^{\ell} k_i\big)$	$O\big((n^3/\varepsilon) \sum_{i=1}^{\ell} k_i\big)$	$\varepsilon = O(d^{-2})$
t-doped Clifford testing	$d = 2$	[GIKL23]	poly(n) if $t = O(\log n)$	poly(n) if $t = O(\log n)$	t-doped Cliffords for $t = O(\log n)$ cannot generate pseudorandom states; Operations over 2 copies
		[GIKL24b]	$O(n)$	$O(n^3)$	t-doped Cliffords for $t < n/2$ cannot generate pseudorandom states; Operations over 2 copies
	$d \geqslant 2$ prime	[ADIS24]	$O(d^{4t})$	$O(nd^{4t})$	t-doped Cliffords for $t = O(\log_d n)$ cannot generate pseudorandom states; Operations over 2 copies
	$d \geqslant 2$	This work	$O(n)$	$O(n^3)$	t-doped Cliffords for $t < n/2$ cannot generate pseudorandom states; Operations over 8 copies
Tolerant stabiliser testing	$d = 2$	[GIKL24b]	$O(1/\gamma^2)$ where $\gamma \triangleq (1-\varepsilon_1)^6 - \frac{4-3\varepsilon_2}{4}$	$O(n/\gamma^2)$	$\varepsilon_1 \leqslant 1 - (1 - 3\varepsilon_2/4)^{1/6}$
		[AD25]	$1/\operatorname{poly}(1-\varepsilon_1)$	$n/\operatorname{poly}(1-\varepsilon_1)$	$1 - \varepsilon_2 \leqslant (1-\varepsilon_1)^{O(1)}$
		[BvDH25]	$1/\operatorname{poly}(1-\varepsilon_1)$	$n/\operatorname{poly}(1-\varepsilon_1)$	$1 - \varepsilon_2 \leqslant (1-\varepsilon_1)^{O(1)}$
		[MT25]	$1/\operatorname{poly}(1-\varepsilon_1)$	$n/\operatorname{poly}(1-\varepsilon_1)$	$1 - \varepsilon_2 \leqslant (1-\varepsilon_1)^{O(1)}$
		[IL24]	$1/\operatorname{poly}(1-\varepsilon_1)$	$n/\operatorname{poly}(1-\varepsilon_1)$	$1 - \varepsilon_2 \leqslant (1-\varepsilon_1)^{O(1)}$ Accepts mixed state inputs
	$d \geqslant 2$	[GNW21]	$O(d^2/\varepsilon)$	$O(nd^2/\varepsilon)$	$\varepsilon_1 = 0$ and $\varepsilon_2 = \varepsilon$ Operations over $2r$ copies, $\gcd(d, r) = 1$
		This work	$O(d^2/\varepsilon)$	$O(nd^2/\varepsilon)$	$\varepsilon_1 = 0$ and $\varepsilon_2 = \varepsilon$ Operations over 8 copies
		This work	$O(r/\gamma_r^2)$ where $\gamma_r \triangleq (1-\varepsilon_1)^{2r} - 1 + \big(1 - (1 - \frac{1}{4d^2})^{r-1}\big)\varepsilon_2$	$O(nr/\gamma_r^2)$	$\varepsilon_1 = O(d^{-2})$ Operations over $2r$ copies, $\gcd(d, r) = 1$
		This work	$O(1/\alpha^2)$ where $\alpha \triangleq (1-\varepsilon_1)^{16}(1-2\varepsilon_1)^4 - \big(1 - \frac{\varepsilon_2}{2d^2}\big(1 - \frac{1}{8d^2}\big)\big)^4$	$O(n/\alpha^2)$	$\varepsilon_1 = O(d^{-2})$ Operations over 8 copies

Table 1: A comparison of our results with several known results in the literature for applications of Bell sampling. In the remarks column, *operations over m copies* indicates that quantum operations are performed on m-fold tensor products of the state under consideration. The prime decomposition of d is $d = \prod_{i=1}^{\ell} p_i^{k_i}$ with $p_1 < p_2 < \cdots < p_\ell$.

- Learning an unknown stabiliser state $|\mathcal{S}\rangle \in (\mathbb{C}^d)^{\otimes n}$ in $O(n^3)$ time with $O(n)$ samples, generalising and/or improving previous works [Mon17, ADIS24];
- Solving the Hidden Stabiliser Group Problem (a stabiliser version of the State Hidden Subgroup Problem) in $\widetilde{O}(n^3/\varepsilon)$ time with $\widetilde{O}(n/\varepsilon)$ samples, improving [HEC25];
- Testing whether $|\psi\rangle \in (\mathbb{C}^d)^{\otimes n}$ has *stabiliser fidelity* at least $1 - \varepsilon_1$ or at most $1 - \varepsilon_2$, using $O(\frac{d^2}{\varepsilon_2})$ samples if $\varepsilon_1 = 0$ or $O(\frac{d^2}{\varepsilon_2^2})$ samples if $\varepsilon_1 = O(d^{-2})$, generalising [GNW21, GIKL24b];
- Testing whether $|\psi\rangle \in (\mathbb{C}^d)^{\otimes n}$ has *stabiliser size* (a generalisation of *stabiliser dimension* for modules) at least d^t, or is ε-far from all such states, in $\widetilde{O}(n^3/\varepsilon)$ time with $\widetilde{O}(n/\varepsilon)$ samples if $\varepsilon = O(d^{-2})$, extending [GIKL25];
- Testing whether $|\psi\rangle \in (\mathbb{C}^d)^{\otimes n}$ is either Haar-random or the output of a Clifford circuit augmented with fewer than $n/2$ single-qudit non-Clifford gates (*doped Clifford circuits*) in $O(n^3)$ time using $O(n)$ samples. As a corollary, we show that Clifford circuits with at most $n/2$ single-qudit non-Clifford gates cannot prepare pseudorandom states, an exponential improvement over previous works on qudits [ADIS24, HEC25], and generalising [GIKL24b].

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